Smoke Simulation

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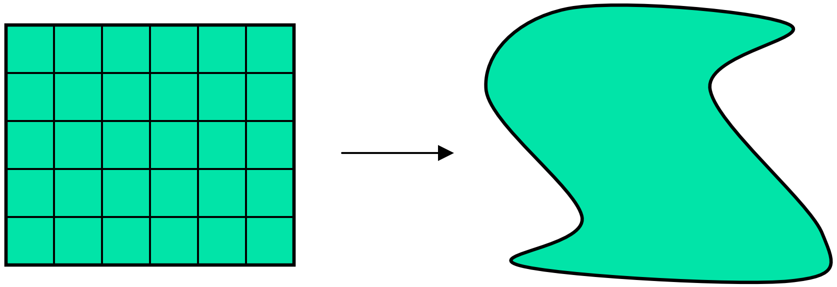
Liquid Vs. Deformable Objects

- Unlike deformable objects, liquid does not retain its original shape
- Using a distortable grid can become too difficult to maintain
- Instead, use a fixed grid, and update the cells in the grid

Euler vs. Lagrange

- Fixed grid
- Points do not change but contents of cells do
- Used for volume rendering or iso-surfaces

- Points are distorted over time



Simulating Smoke

- Smoke is essentially particles moving in air
- Air, like liquid, is a medium
- Therefore we can apply the same theory of simulating liquid for simulating smoke

Euler Equations

- (1) Conservation of mass:

$$\nabla \bullet u = 0$$

  - u : velocity vector of gas as function u(x,t) of space (x) and time (t)

- (2) Conservation of momentum:

$$\frac{\partial u}{\partial t} = -(u \bullet \nabla)u - \nabla p + f$$

  - f : external forces
  - p : pressure of the gas

Solving Equations

- Compute an intermediate velocity field u* by solving Equation 2 over a time step Δt. The pressure term p is left out.

$$\frac{u^* - u}{\Delta t} = \underbrace{-(u \bullet \nabla)}_{\text{Advection/convection operator}} u + f$$

Solving Equation (cont'd)

$$\frac{u^* - u}{\Delta t} = -(u \bullet \nabla)u + f$$

$$u^* = (-(u \bullet \nabla)u + f)\Delta t + u$$

$$u^* = f\Delta t - (u \bullet \nabla)u\Delta t + u$$

Use semi-Langrangian method to solve advection operator

Satisfying Equation 1

- We force the field u* to be incompressible (aka satisfying $\nabla \bullet u = 0$) by computing the pressure p from the Poisson equation

$$\nabla^2 p = \frac{1}{\Delta t} \nabla \bullet u^*$$

- Force $\nabla \bullet u = 0$

Solving final velocity

- u* becomes incompressible after subtracting the gradient of the pressure from it.

$$u = u^* - \Delta t \nabla p$$

Solving final velocity (cont'd)

$$u = u^* - \Delta t \nabla p$$

$$\nabla \bullet u = \nabla \bullet (u^* - \Delta t \nabla p)$$

$$\nabla \bullet u = \nabla \bullet u^* - \nabla \bullet (\Delta t \nabla p)$$

$$\nabla \bullet u = \nabla \bullet u^* - \Delta t \nabla^2 p$$

$$\nabla \bullet u = 0$$

Density equation

- Equation for change of smoke density over time:

$$\frac{dp}{dt} = 0 \quad \Rightarrow \quad \frac{\partial p}{\partial t} = -(u \bullet \nabla)\, p$$

- Density change is supposed to be 0

Taking temperature to account

- Temperature and density affects the smoke velocity u. Forces that are applied to smoke:
  - Downward force: Gravity
  - Upward force: buoyancy, directly related to temperature

$$f_{buoy} = -\alpha p\vec{z} + \beta(T - T_{amb})\vec{z}$$

alpha, beta = constants

p=density

z= up vector (0,0,1)

T=temperature

T_{amb}= ambient temperature of the air

Demo

- Smoke demonstration was taken from:
  - R. Fedkiw, J. Stam, and H.W. Jensen, *Visual Simulation of Smoke*, Proceedings of SIGGRAPH`2001