

AMG on the GPU

Exposing fine-grained parallelism

Copper 2011

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Motivation 1: potential

1M d.o.f.
10 nonzero neighbors
1.5 cycle complexity
15M computations

2 for a $V(1,1)$ -cycle
20 iterations
600M for solution

16 bytes for (data,row,col)
10 Gbytes of data to process

125 Gbyte/sec SpMV on the GPU?
~ **0.08 sec**

25 Gbyte/sec SpMV on host?
~ **0.4 sec**

Motivation 2: software + hardware

real acceleration units: dedicated computation, double precision, high speeds

useable software: CUDA + Thrust + Cusp

AMG “asks” for acceleration:

- ✓ adaptive
- ✓ multiple candidates, cycles
- ✓ less/more aggressive coarsening
- ✓ multiple RHS

I. some basics about the software

II. some details about parallelism

III. some results on performance

goal: an efficient, useable, extensible AMG
algorithm on the GPU

Application

CURAND

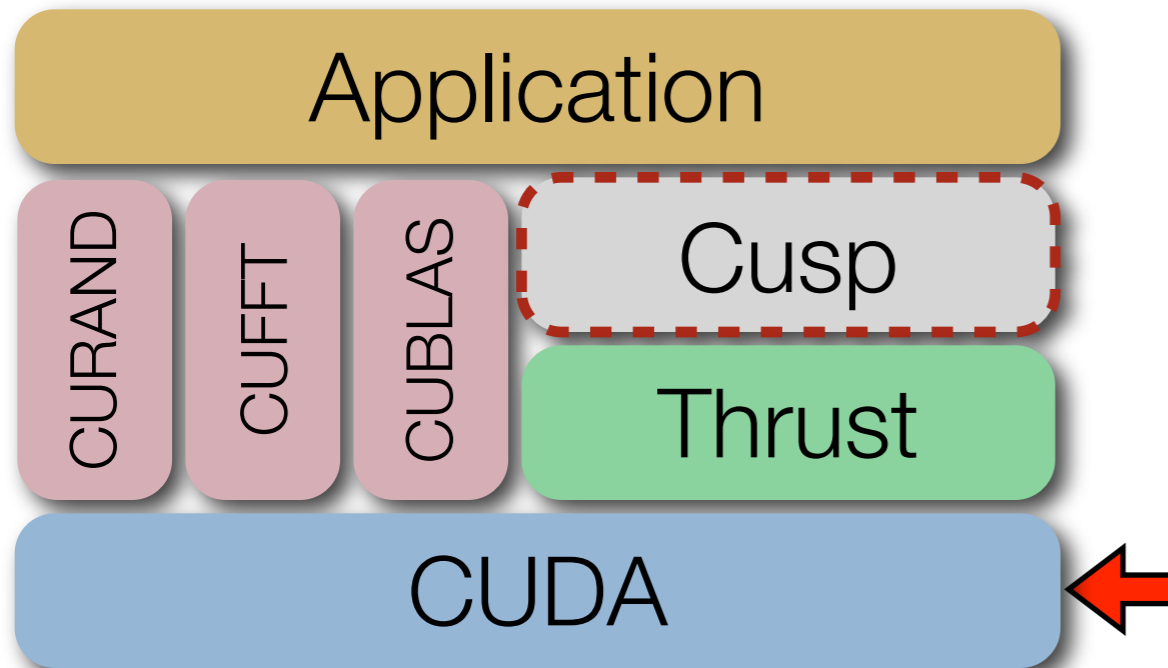
CUFFT

CUBLAS

Cusp

Thrust

CUDA



```
__global__ void function(float *a)
{
    // perform action on device
}

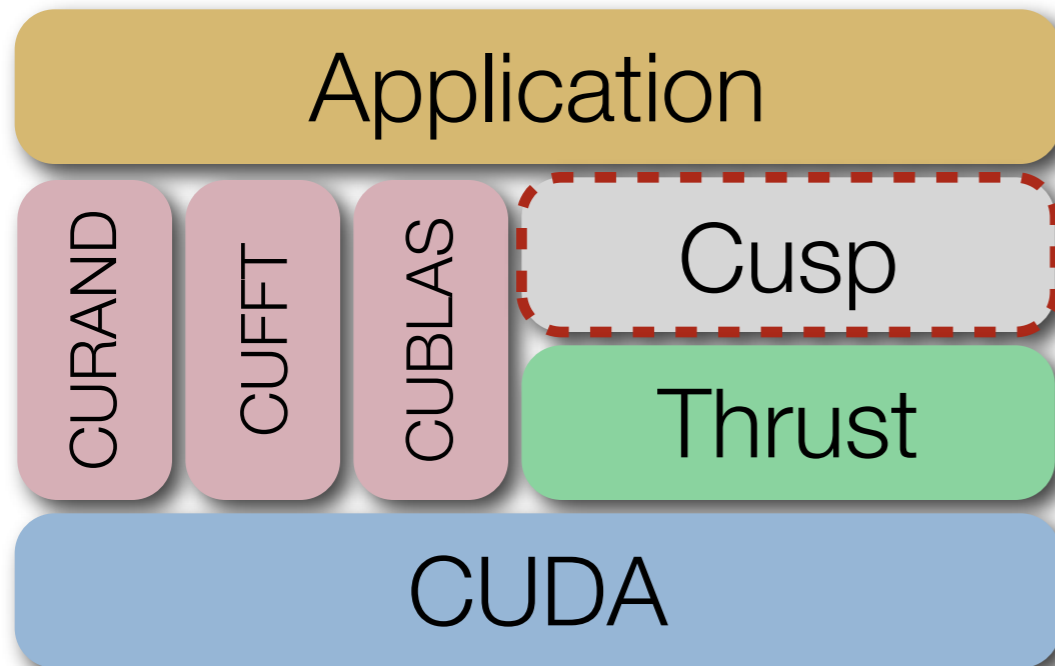
int main( int argc, char * argv[] )
{
    float *a_h, *a_d;
    float *b_h;
    int N = 10000;
    size_t size = N * sizeof(float);
    a_h = (float *)malloc(size); // allocate size bytes on host
    cudaMalloc((void **) &a_d, size); // allocate same on device

    // initialize a_h

    // copy data to device
    cudaMemcpy(a_d, a_h,
               sizeof(float)*size, cudaMemcpyHostToDevice);

    int blockSize = 4; // threads per block
    int nBlocks = N/blockSize + ...; // # of blocks needed
    incrementArrayOnDevice <<< nBlocks, blockSize >>> (a_d);
    cudaMemcpy(b_h, a_d,
               sizeof(float)*N, cudaMemcpyDeviceToHost);

    free(a_h); free(b_h); cudaFree(a_d); // deallocate
}
```



- fast development
- low overhead
- open source

```
#include <thrust/host_vector.h>
#include <thrust/device_vector.h>
#include <thrust/sort.h>
#include <cstdlib.h>

int main(void)
{
    // generate 32M random numbers on the host
    thrust::host_vector<int> h_vec(32 * 1024 * 1024);
    thrust::generate(h_vec.begin(), h_vec.end(), rand);

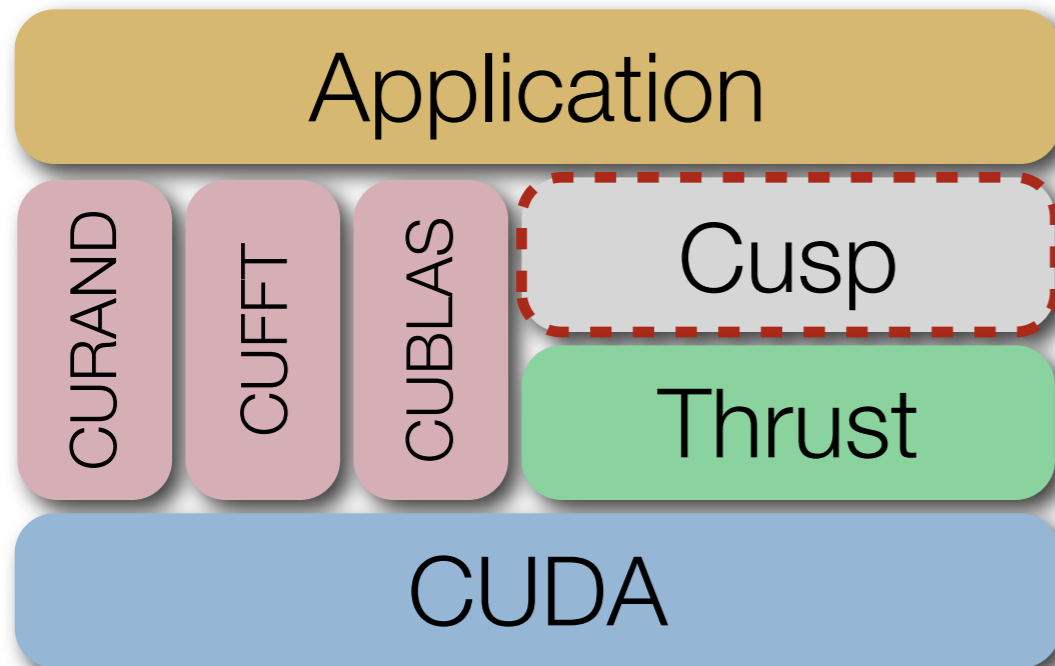
    // transfer data to the device
    thrust::device_vector<int> d_vec = h_vec;

    // sort data on the device (846M keys per sec on GeForce GTX
    480)
    thrust::sort(d_vec.begin(), d_vec.end());

    // transfer data back to host
    thrust::copy(d_vec.begin(), d_vec.end(), h_vec.begin());

    return 0;
}
```

- like C++ STL for CUDA
- usability
- containers and algorithms on host and device



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- low overhead
- open source

```
#include <culp/hyb_matrix.h>
#include <culp/io/matrix_market.h>
#include <culp/krylov/cg.h>

int main(void)
{
    // create an empty sparse matrix structure (HYB format)
    culp::hyb_matrix<int, float, culp::device_memory> A;

    // load a matrix stored in MatrixMarket format
    culp::io::read_matrix_market_file(A, "5pt_10x10.mtx");

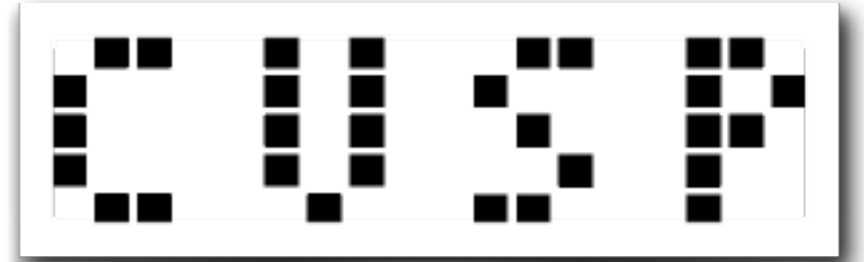
    // allocate storage for solution (x) and right hand side (b)
    culp::array1d<float, culp::device_memory> x(A.num_rows, 0);
    culp::array1d<float, culp::device_memory> b(A.num_rows, 1);

    // solve the linear system A * x = b
    // with the Conjugate Gradient method
    culp::krylov::cg(A, x, b);

    return 0;
}
```

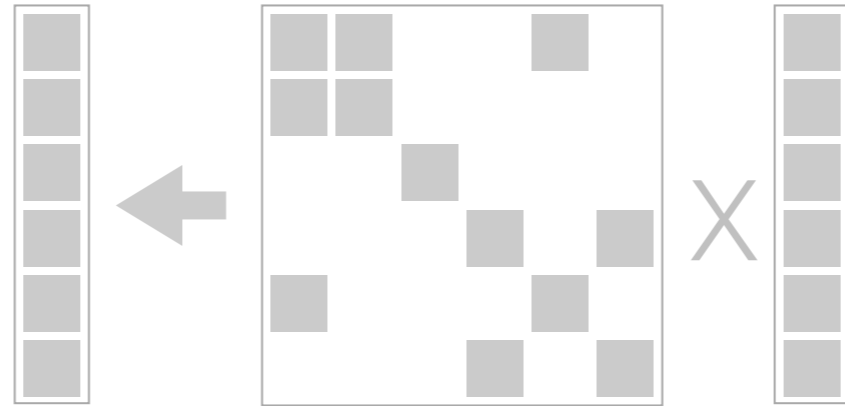
- sparse matrix support
- solvers
- io
- “views” of other memory

Cusp



- Sparse matrix containers: COO, CSR, DIA, ELL, HYB
- Sparse Matrix-Vector multiply (SpMV)
- Sparse Matrix-Matrix multiply (SpMM)
- Transpose and format conversions
- Maximal independent sets
- Solvers

SpMV



- *memory-bound*

- low arithmetic intensity (flop/mem. transfer)

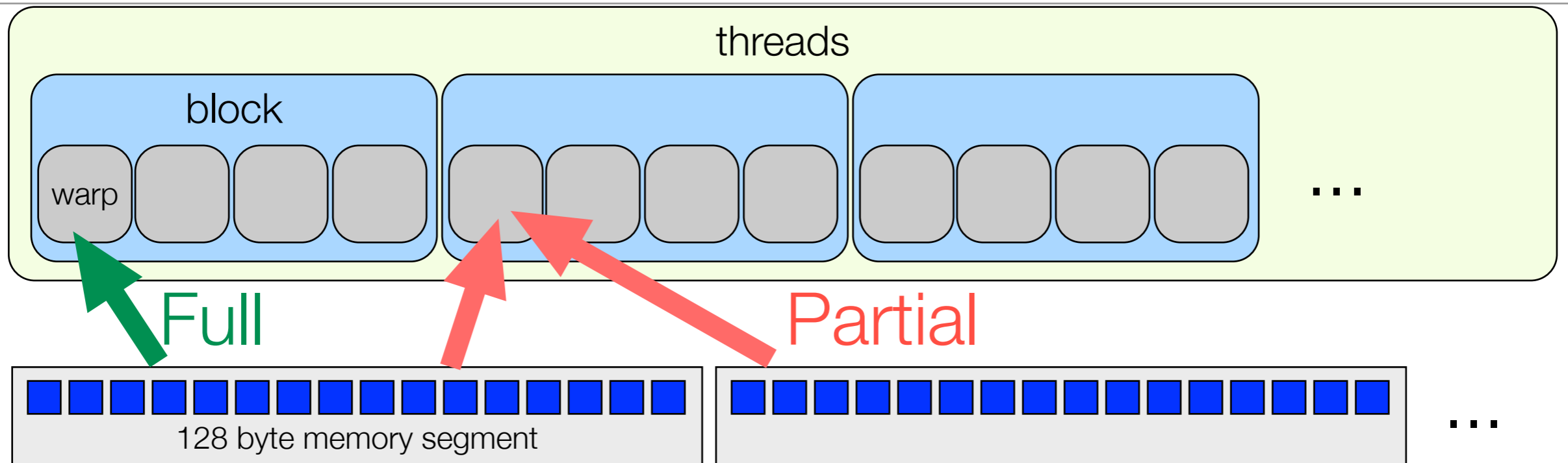
- Tesla C2050 peak (double):

515 GFLOP/s performance
144 GB/s bandwidth
3.57 FLOP/byte

- Typical SpMV:

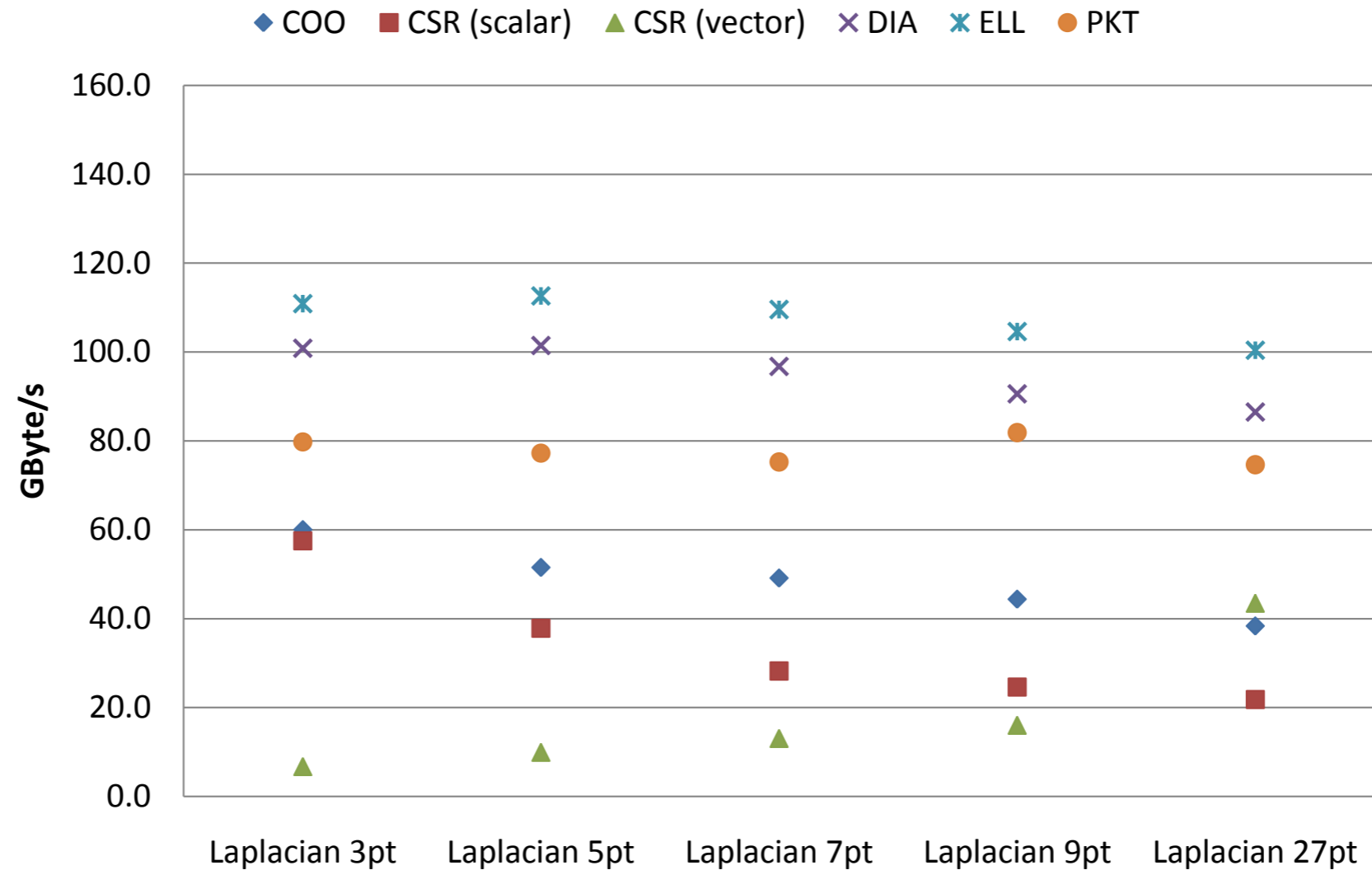
5-20 GFLOP/s performance
100 GB/s bandwidth
0.1 FLOP:bytes (2:16)

SpMV: memory coalescing and parallelism



- **DIA, ELL, CSR(scalar)** } full, needs many rows
 - one thread per row
- **CSR(vector)** } partial, few rows sufficient
 - one warp per row
- **COO** } low, insensitive to # of rows
 - one thread per nonzero

SpMV results



- Summary: structure formats do better with structure
- Results are often a toss-up as structure is lost (think coarse grids)

AMG Components: what we need

- a strength routine

$$S_{ij}$$

- a threaded aggregation method

$$S \rightarrow Agg$$

- an interpolation assembly

$$B, Agg \rightarrow T$$

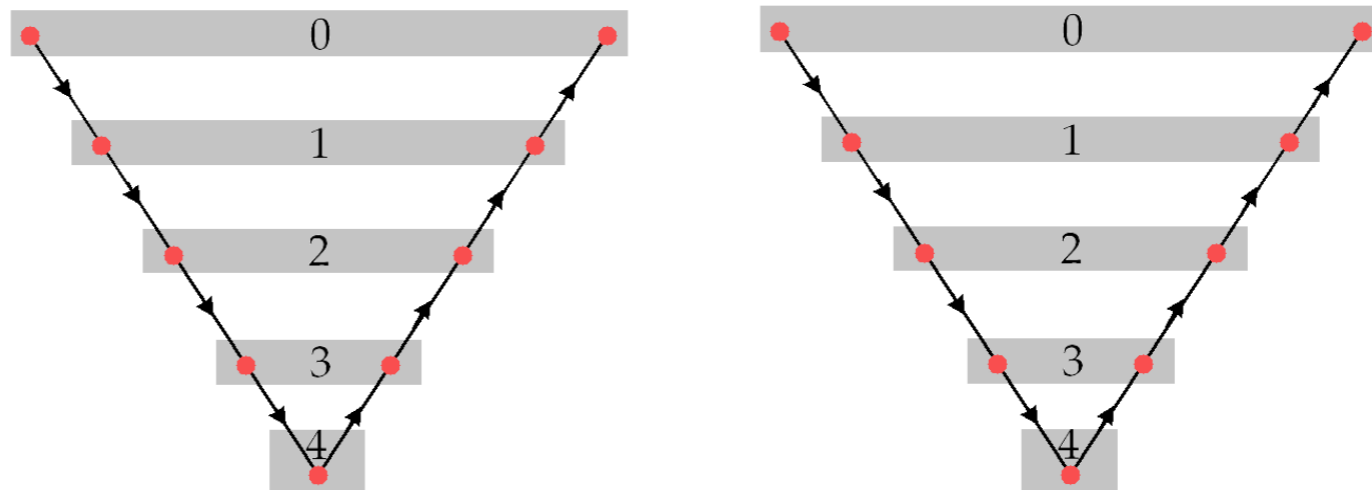
- smoothing interpolation

$$T \rightarrow P$$

- coarse-grid operator

$$P^T AP$$

- cycling



Setup

Solve

Strength-of-connection

parameters: A

return: S

$D \leftarrow 0$

for $n \in [0, nnz]$ **do**

if $I_n = J_n$
 $D(I_n) = V_n$
 if $|V_n| > \theta \sqrt{|D(I_n)| \cdot |D(J_n)|}$
 $S_{ij} = 1$

- simplifies to stream reduction (contraction)

- use `thrust::copy_if(V, V + N, result, is_even());`

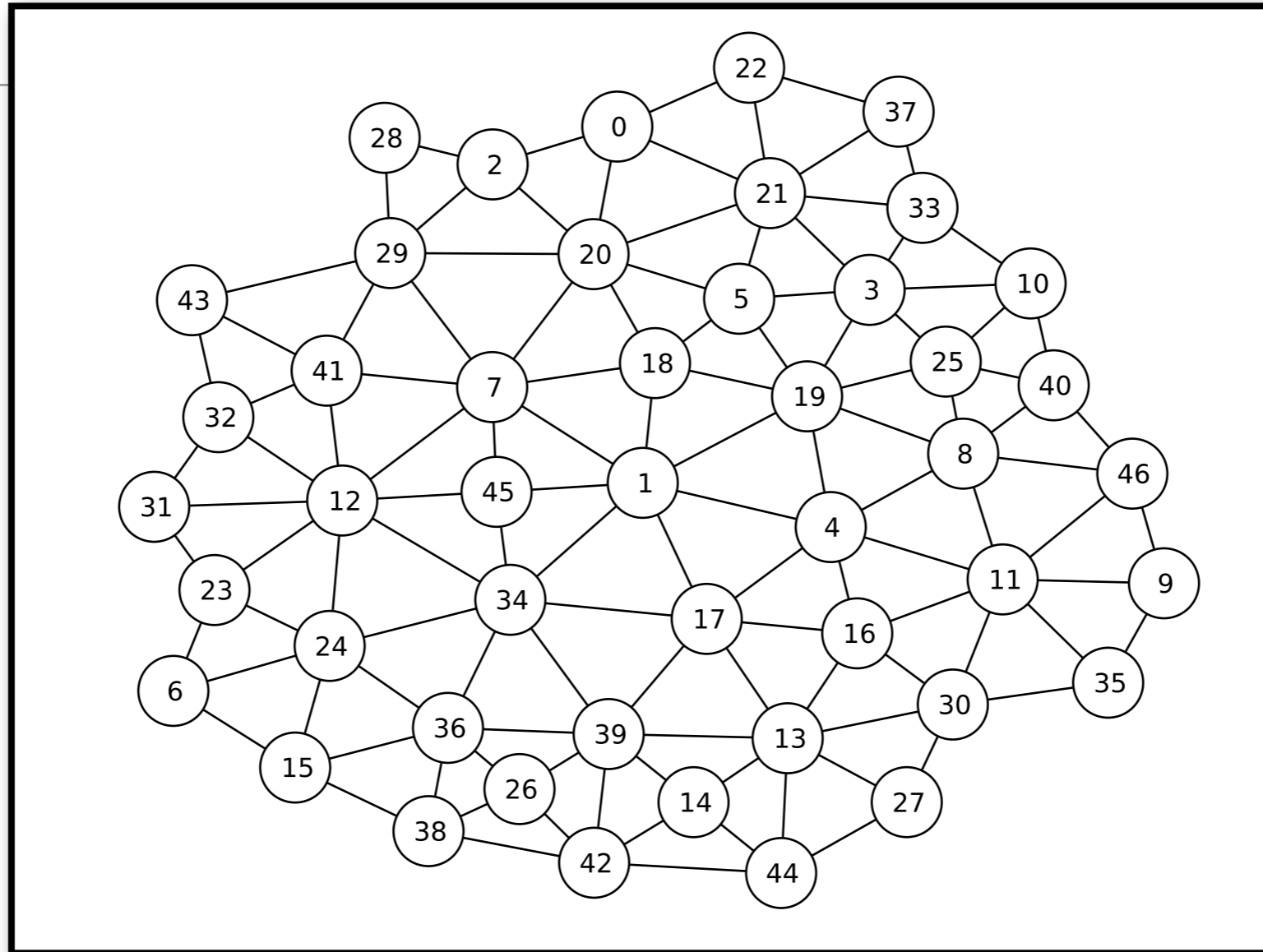
Aggregation

- standard aggregation: greedy, sequentially dependent
- “new” threaded aggregation: generalized modified independent sets

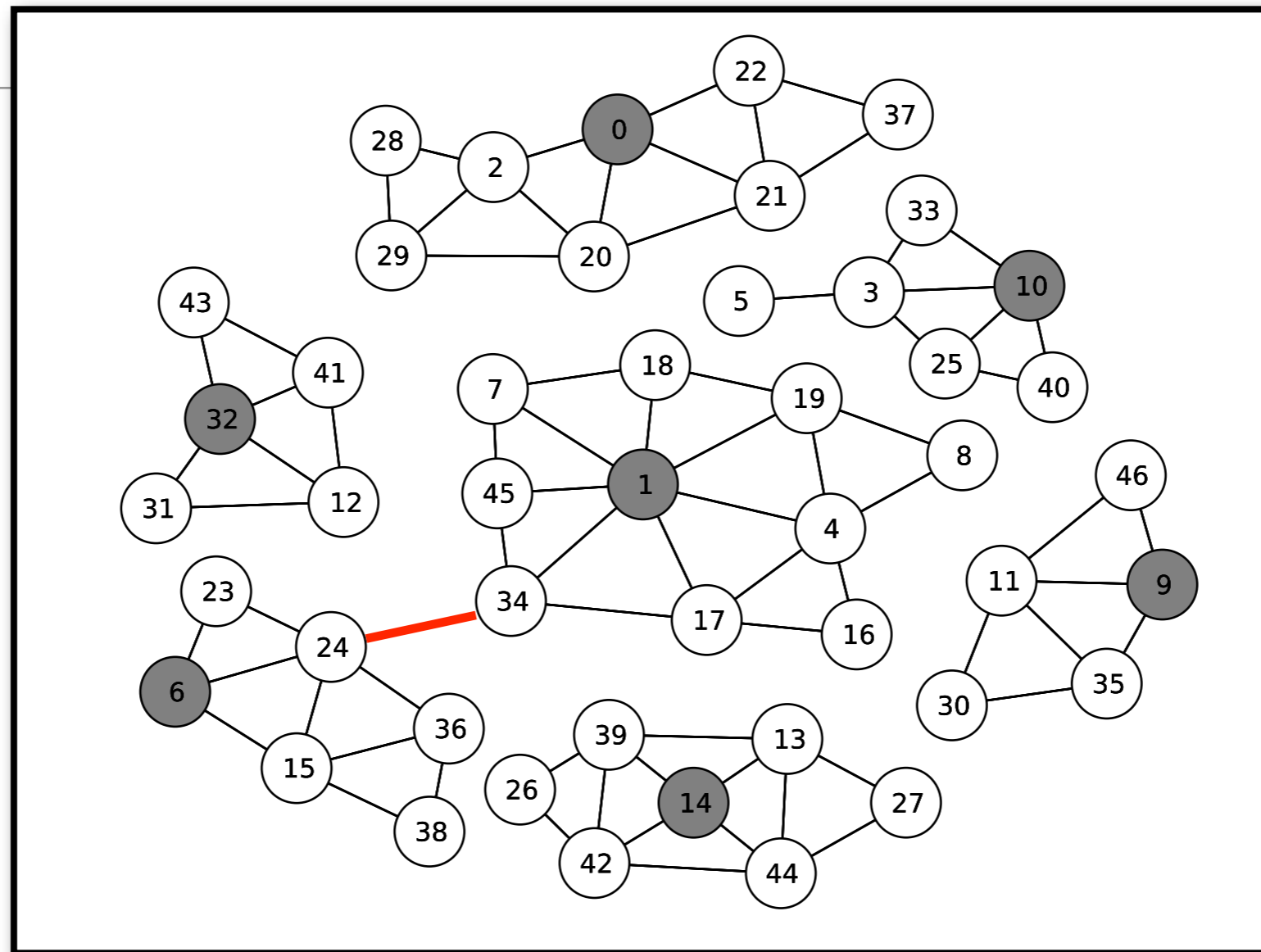
standard = threaded
(up to matrix permutation)

- memory access and parallelism resembles SpMV

MIS(2)



MIS(2)



independent

- root nodes more than 2 edges apart ($> \text{distance}-2$)
- an unaggregated node more than 2 edges from a root can become a root

MIS(2)

maximal

MIS(2)

1. Given a MIS(2) of $\{0,1\}$ (N nodes)

2. prefix scan to enumerate

```
thrust::exclusive_scan(set, set + N, set, init)
```

3. communicate to neighbors
aggregate index (**SpMV**)

~ std. first pass

4. communicate index to unaggregated
neighbors (another **SpMV**)

~ std. second pass

- Generalizes to MIS(k)
- allows for variable coarsening

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Transpose, Smoothing, spectral radius

- Smoothing: all **SpMV**s with GS or wJ
- Spectral radius: all **SpMV**s with Arnoldi^{***}
- Transpose: **sort_by_key** (millions integer keys/sec)

std::sort	tbb:parallel_sort	thrust::sort
10.6	35.1	804.8

- keys: column index
- values: (row,data) tuple

SpMM

- SMMP algorithm: very sequential
 - requires $O(\text{ncol})$ storage to determine entries of each sparse row
 - parallelism would require $O(\text{ncol})$ memory per thread

- Consider $C = A * B$

$$A = \begin{bmatrix} 5 & 10 & 0 \\ 15 & 0 & 20 \end{bmatrix}, = \begin{bmatrix} (0, 0, 5) \\ (0, 1, 10) \\ (1, 0, 15) \\ (1, 2, 20) \end{bmatrix}, \quad B = \begin{bmatrix} 25 & 0 & 30 \\ 0 & 35 & 40 \\ 45 & 0 & 50 \end{bmatrix}, = \begin{bmatrix} (0, 0, 25) \\ (0, 2, 30) \\ (1, 1, 35) \\ (1, 2, 40) \\ (2, 0, 45) \\ (2, 2, 50) \end{bmatrix},$$

1. form intermediate view of C
2. sort C by row, col
3. contract C by summing duplicates

SpMM

$$A = \begin{bmatrix} 5 & 10 & 0 \\ 15 & 0 & 20 \end{bmatrix}, B = \begin{bmatrix} 25 & 0 & 30 \\ 0 & 35 & 40 \\ 45 & 0 & 50 \end{bmatrix},$$

- expand with $A(i, j) * B(i, :)$

$$C = \begin{bmatrix} (0, 0, 125) \\ (0, 2, 150) \\ (0, 1, 350) \\ (0, 2, 400) \\ (1, 0, 375) \\ (1, 2, 450) \\ (1, 0, 900) \\ (1, 2, 1000) \end{bmatrix}$$

- all parallel gather, scatter, scan

```
thrust::gather(map, map + 10, input, out.begin())  
thrust::scatter(input, input + 10, map, out.begin())  
thrust::inclusive_scan(data, data + 6, data)
```

- sort by column keys

$$C = \begin{bmatrix} (0, 0, 125) \\ (0, 1, 350) \\ (0, 2, 150) \\ (0, 2, 400) \\ (1, 0, 375) \\ (1, 0, 900) \\ (1, 2, 450) \\ (1, 2, 1000) \end{bmatrix}$$

```
thrust::stable_sort(A, A + N);
```

SpMM

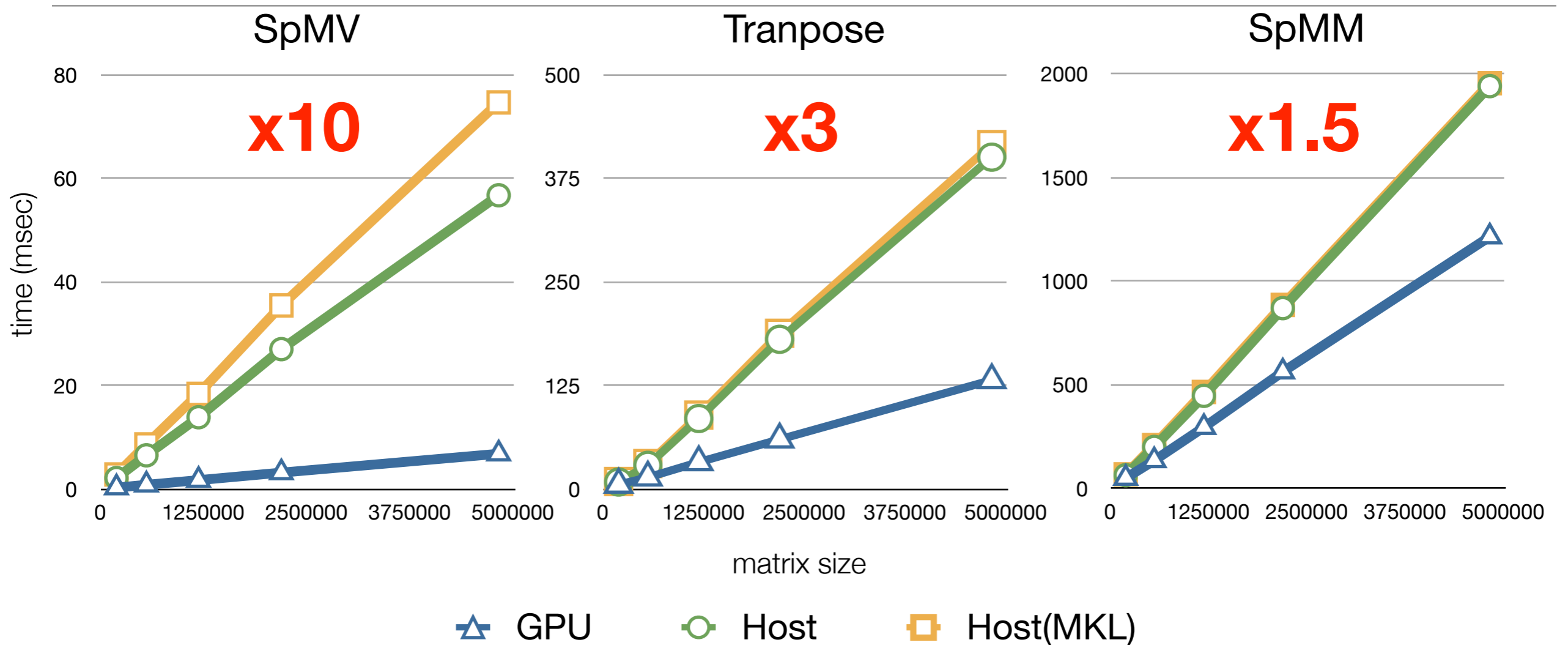
- parallel reduction

$$C = \begin{bmatrix} (0, 0, 125) \\ (0, 1, 350) \\ (0, 2, 550) \\ (1, 0, 1275) \\ (1, 2, 1450) \end{bmatrix} = \begin{bmatrix} 125 & 350 & 550 \\ 1275 & 0 & 1450 \end{bmatrix} \cdot$$

```
thrust::reduce_by_key(A, A + N, B, C, D)
```

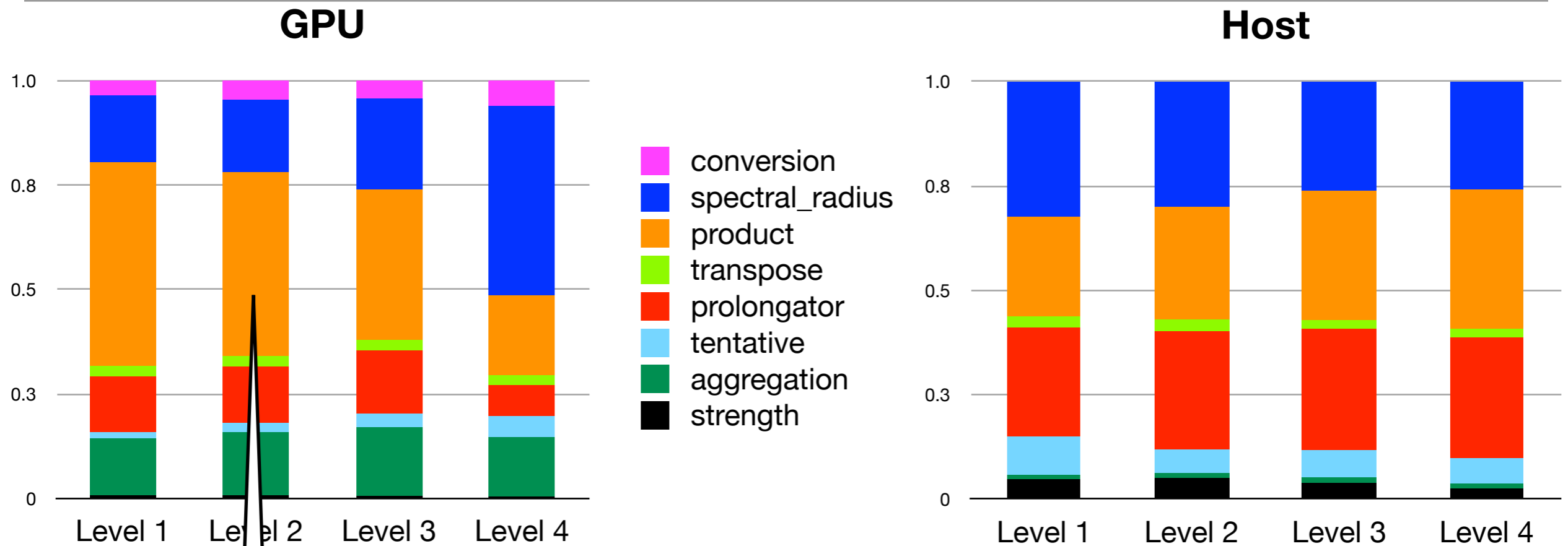
- insensitive to irregularity of input
- same “work” as SMMP
- storage cost can be large for intermediate (reduce by subdividing)

Setup kernels: SpMM, Transpose, SpMV



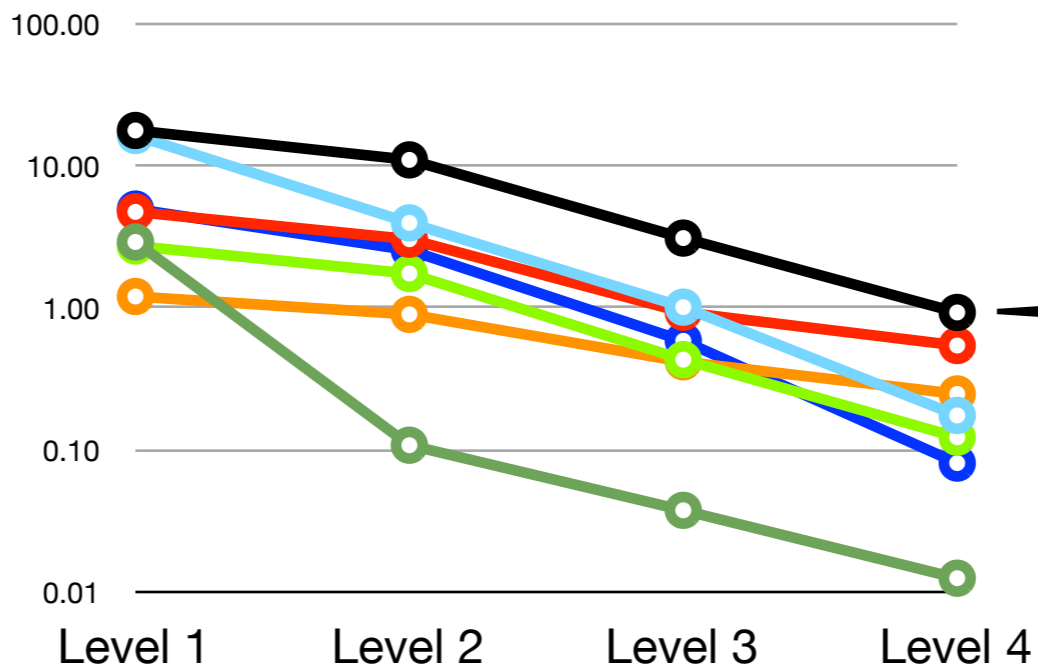
speed up	size	A*B	BT*(A*B)	BT*A*B	Tranpose	SpMV
	5E+06	1.8	1.4	1.6	3.2	10.8
	2E+06	1.7	1.4	1.6	3.1	10.9
	1E+06	1.7	1.3	1.6	2.7	10.1
	6E+05	1.7	1.3	1.5	2.1	9.5
	2E+05	1.6	1.0	1.4	1.6	7.9

Setup components: total time



Triple products are expensive

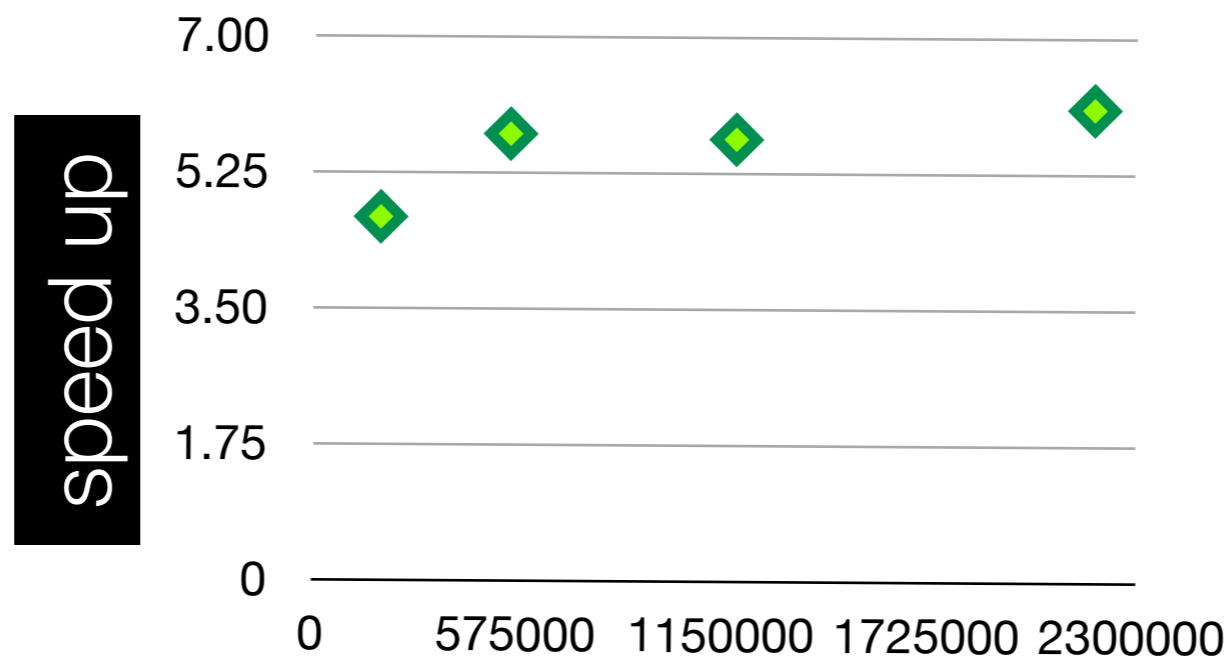
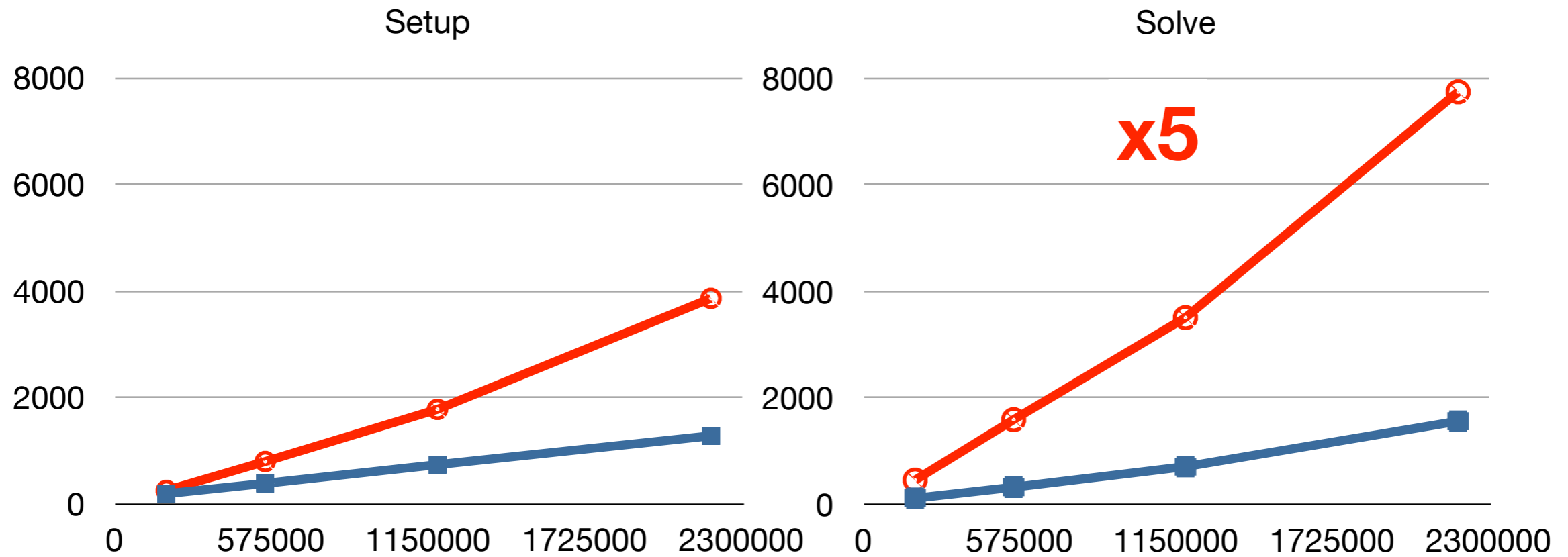
speed up



GPU likes fine (sparse) levels

Solve

+ GPU ○ Host



Summary

exposed SA to **GPU kernels**

tangible speedups

identified **practical** directions for AMG to take
advantage of fine grained parallelism



Sparse matrix conversion times (msec)

- sort, gather, scatter, scan

unstructured

2D	From\To	COO	CSR	ELL	HYB
1M dof	COO	5	6	20	24
8M nnz	CSR	8	4	22	25
	ELL	17	18	6	22
	HYB	63	69	83	4

3D	From\To	COO	CSR	ELL	HYB
1M+ dof	COO	10	12	57	56
17M nnz	CSR	15	8	60	59
	ELL	72	74	19	71
	HYB	139	152	196	10