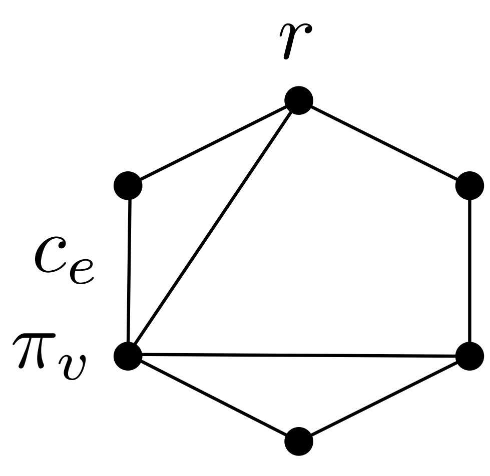
Prize Collecting Steiner Tree and Forest in Planar Graphs

Chandra Chekuri Alina Ene Nitish Korula

Midwest Theory Day 2010

PC Steiner Tree and Forest

(Rooted) Prize-collecting Steiner tree



PC Steiner Tree and Forest

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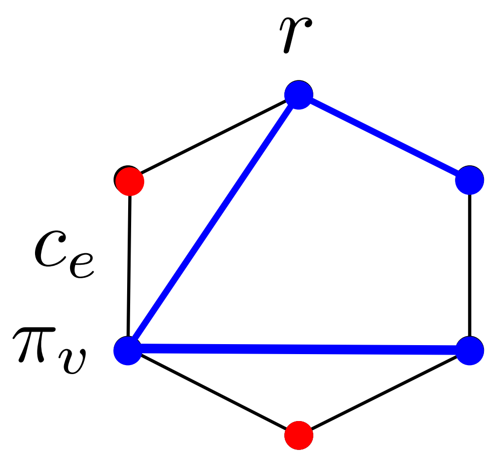


$\text{Length}(T) = \sum_{e \in T} c_e$

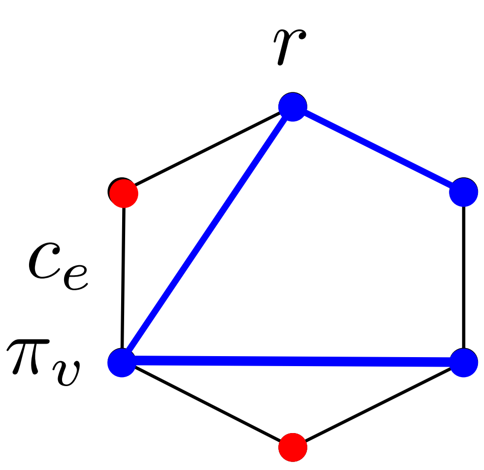
$\text{Penalty}(T) = \sum_{v \notin T} \pi_v$

$\text{Cost}(T) = \text{Length}(T) + \text{Penalty}(T)$

Generalizes Steiner tree

PC Steiner Tree and Forest

(Rooted) Prize-collecting Steiner tree



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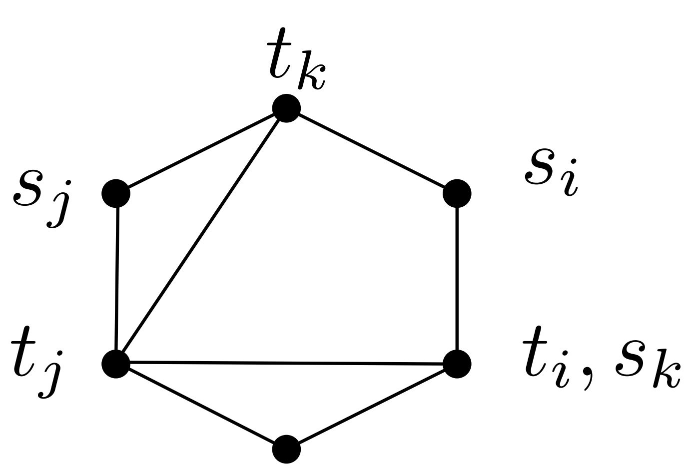

PC Steiner Tree and Forest

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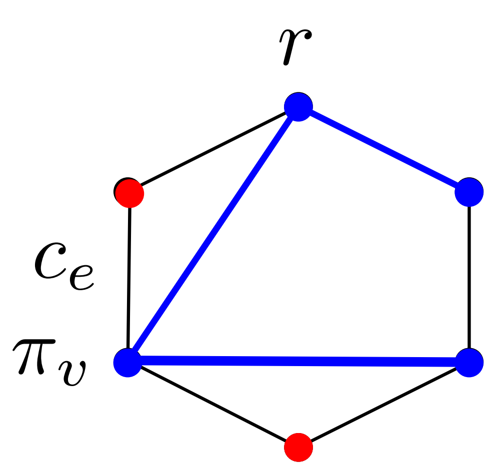

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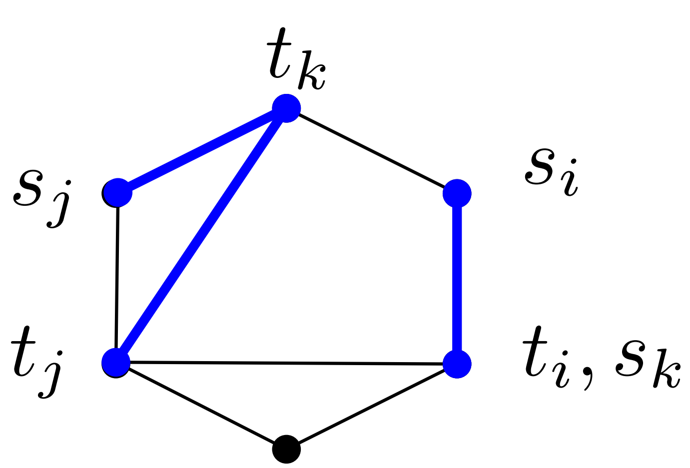

$\text{Length}(F) = \sum_{e \in F} c_e$

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Generalizes Steiner forest

Previous Results

Prize collecting Steiner tree

1.92-approximation (Archer *et al.* 2009)

2-approximation (Goemans-Williamson 1995)

Steiner tree in planar graphs

PTAS (Borradaile-Klein-Mathieu 2009)

Prize collecting Steiner forest

2.54-approximation (Jain-Hajiaghayi 2006)

Steiner forest in planar graphs

PTAS (Bateni-Hajiaghayi-Marx 2010)

Our Results

Prize Collecting Steiner Tree in planar graphs

- PTAS

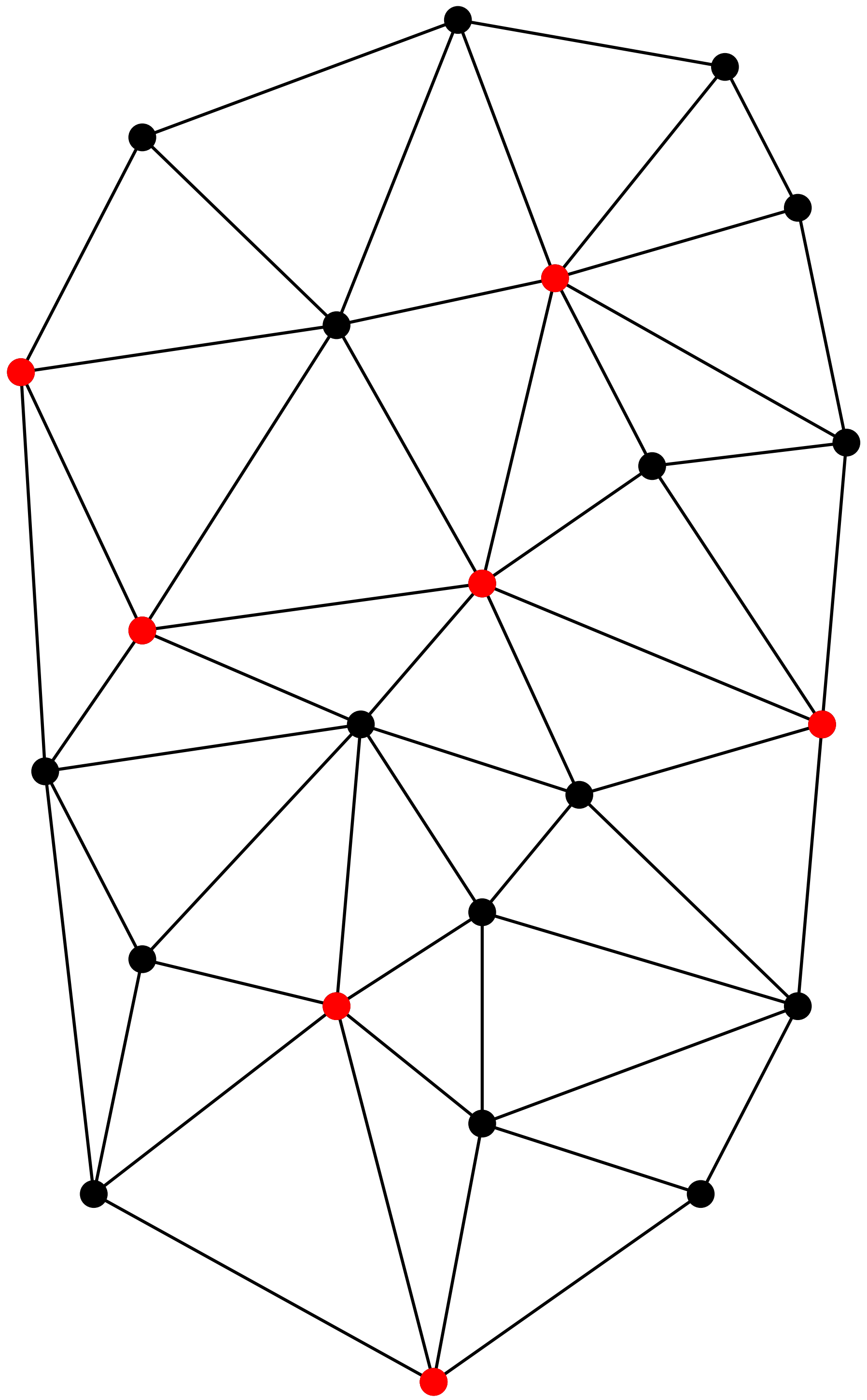
Prize Collecting Steiner Forest in planar graphs

- Reduction to bounded treewidth
- APX-hard on series parallel graphs

  (Bateni-Hajiaghayi-Marx 2010)

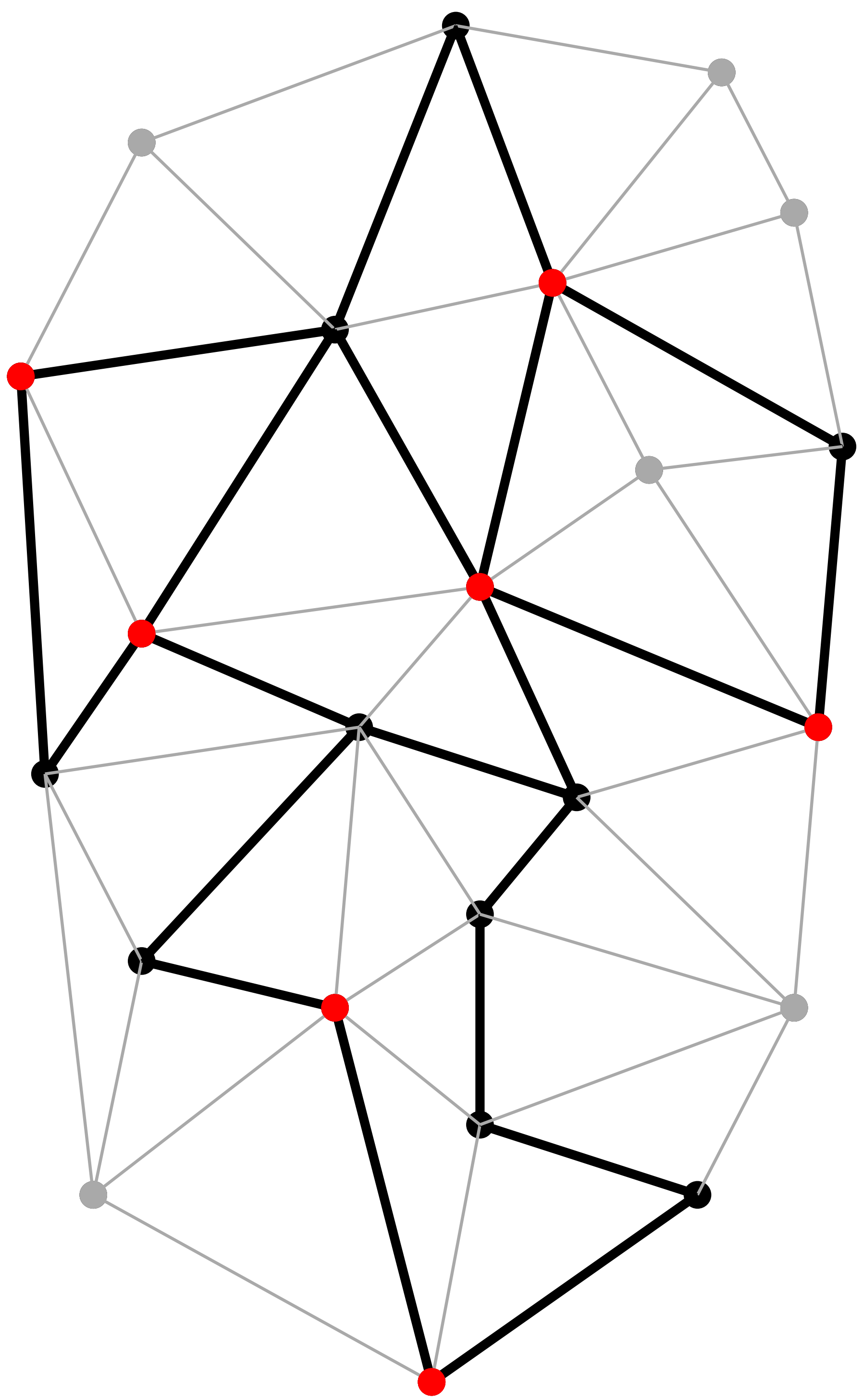
Steiner Tree in Planar Graphs

(Borradaile-Klein-Mathieu 2009)



Steiner Tree in Planar Graphs

(Borradaile-Klein-Mathieu 2009)



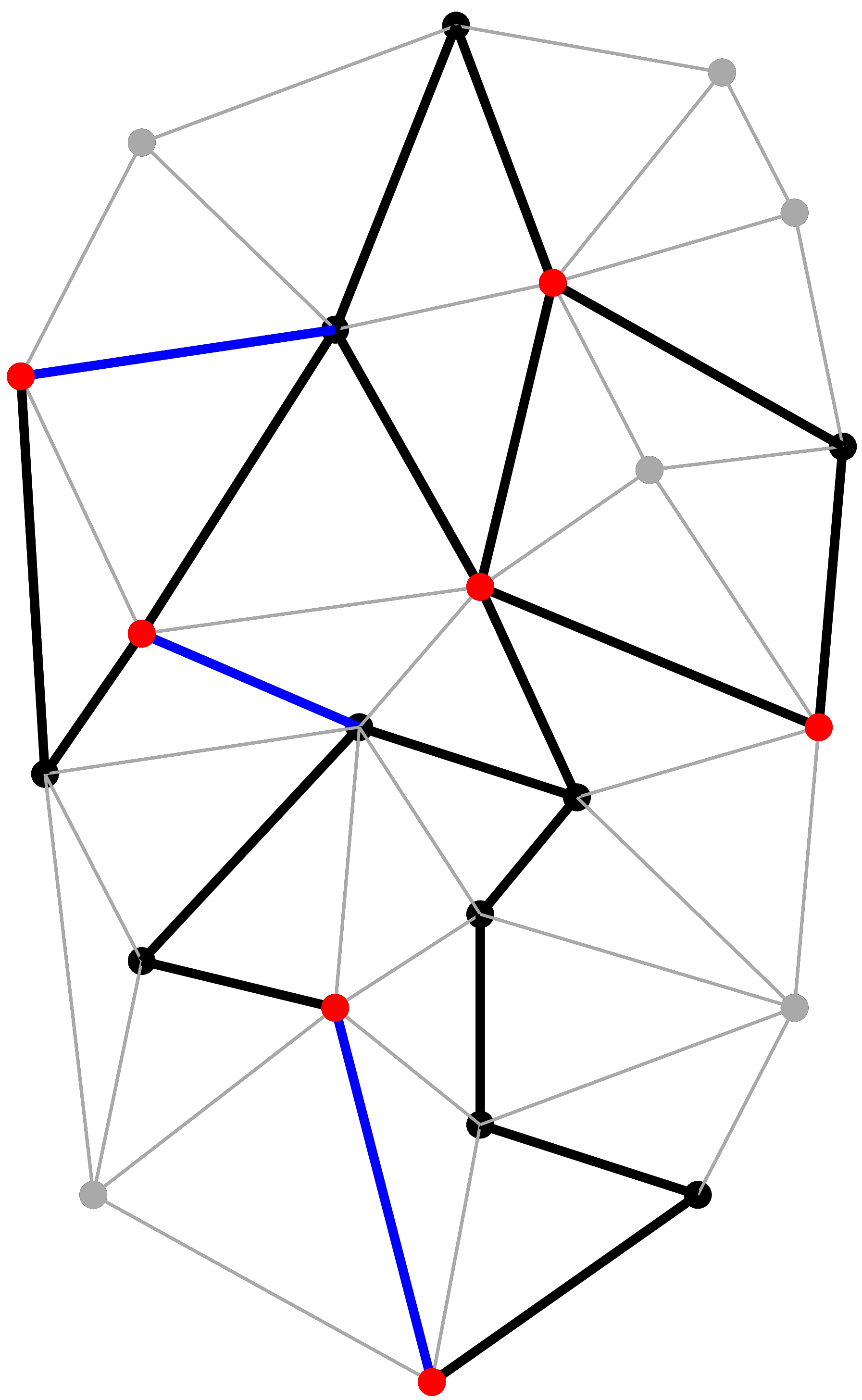
Find a special subgraph spanning the terminals

Length$(H) \leq f(\epsilon)$OPT, OPT$(H) \leq (1+\epsilon)$OPT

Reduce to bounded treewidth

Steiner Tree in Planar Graphs

(Borradaile-Klein-Mathieu 2009)



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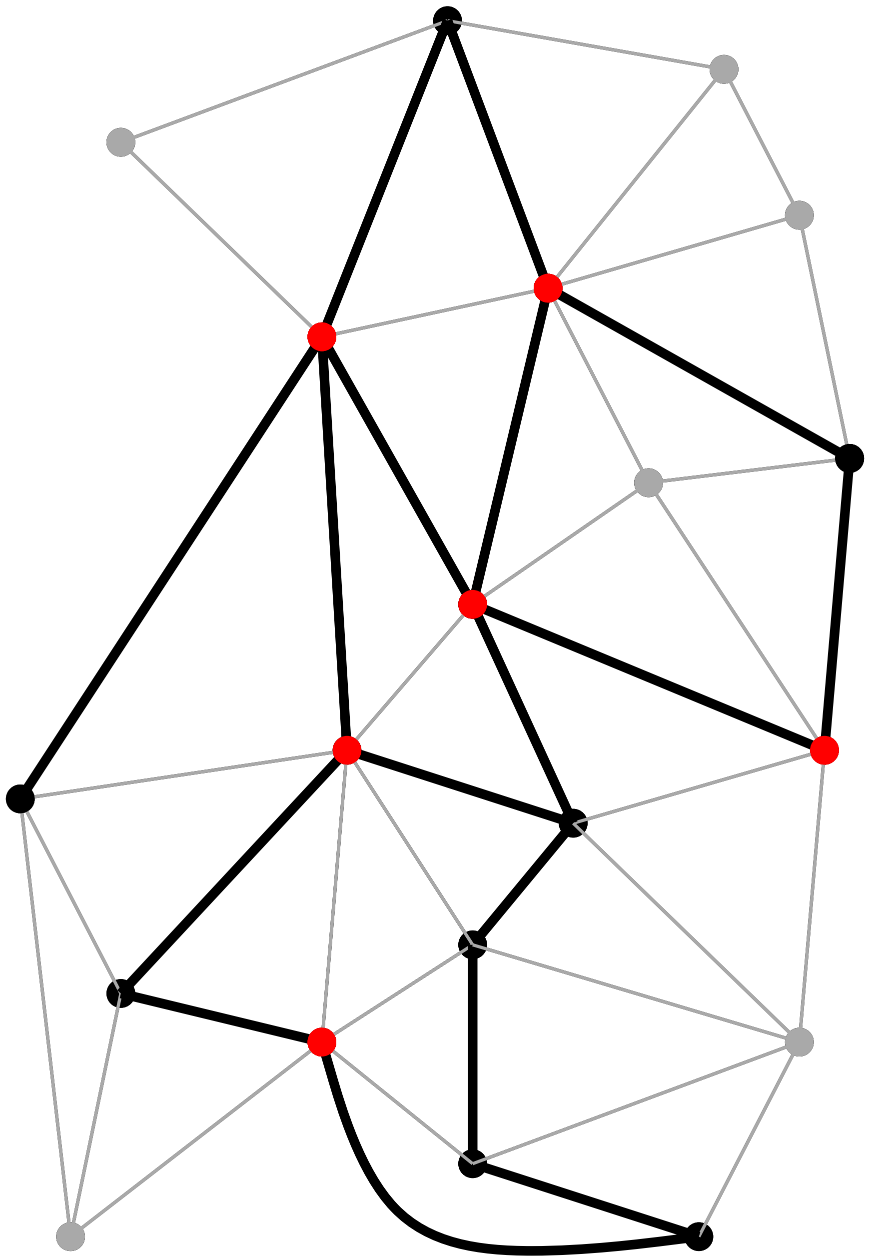
Reduce to bounded treewidth

Light set of edges F: Length$(F) \leq \epsilon$OPT

Steiner Tree in Planar Graphs

(Borradaile-Klein-Mathieu 2009)



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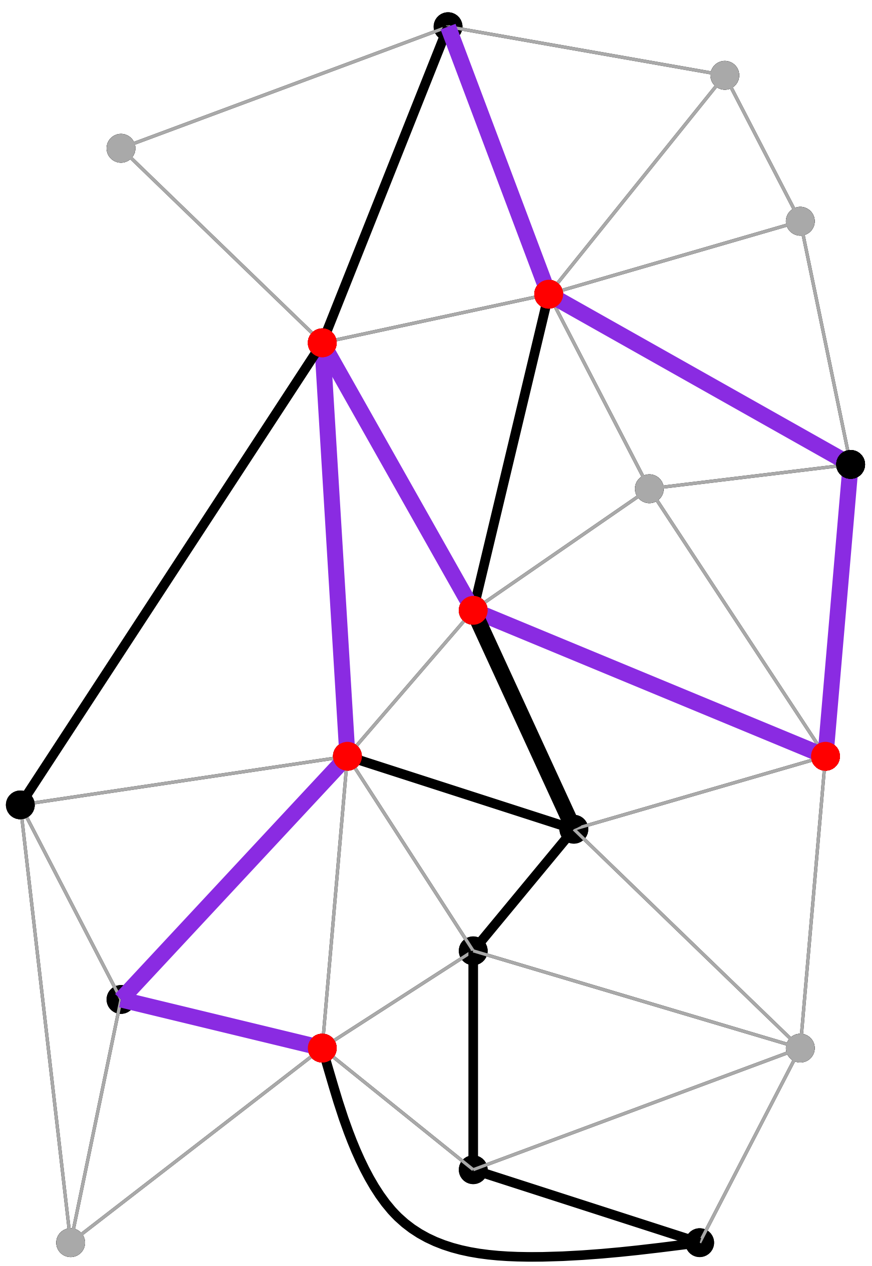
G/F has bounded treewidth

Steiner Tree in Planar Graphs

(Borradaile-Klein-Mathieu 2009)



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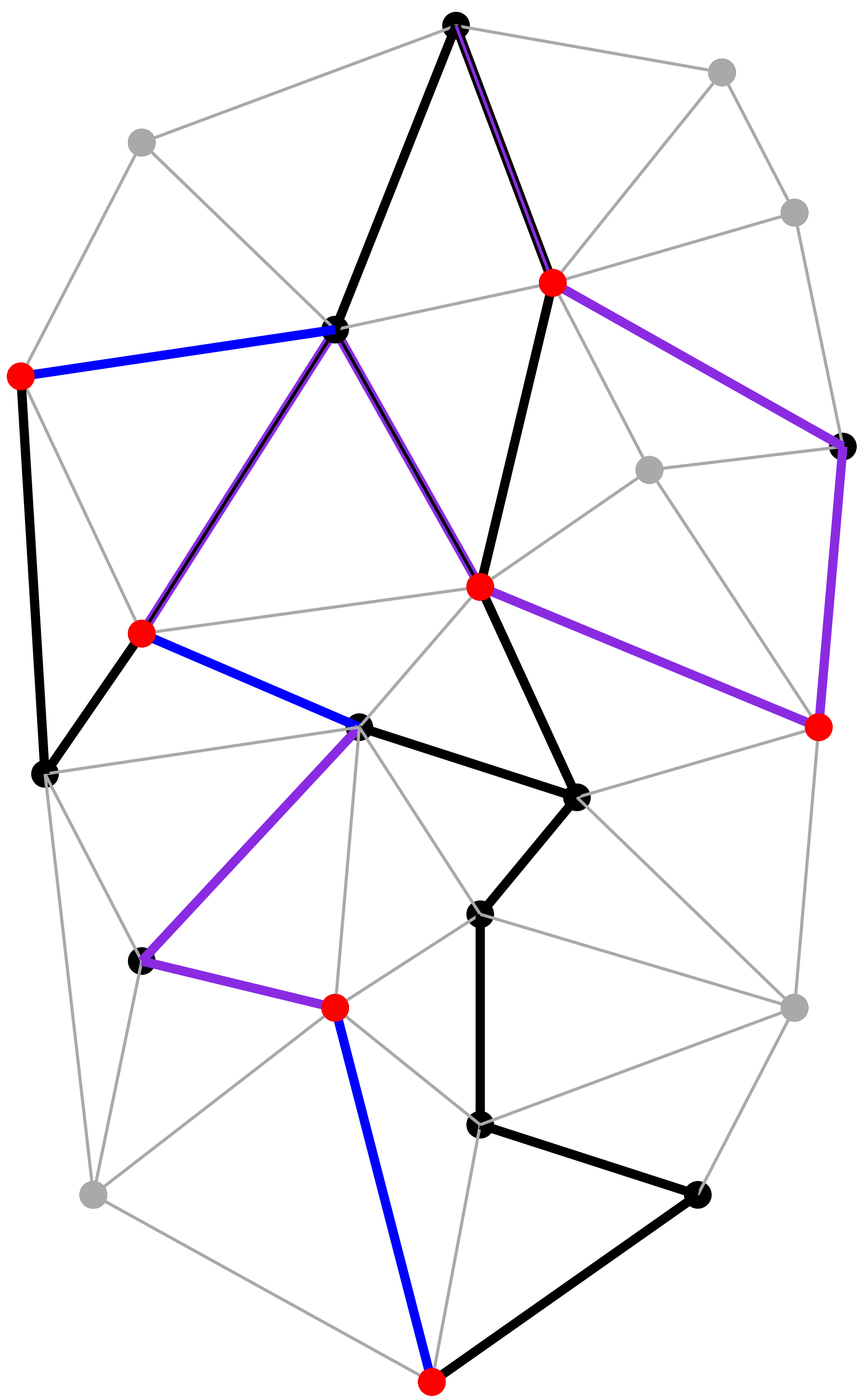
Exact solution T

Steiner Tree in Planar Graphs

(Borradaile-Klein-Mathieu 2009)



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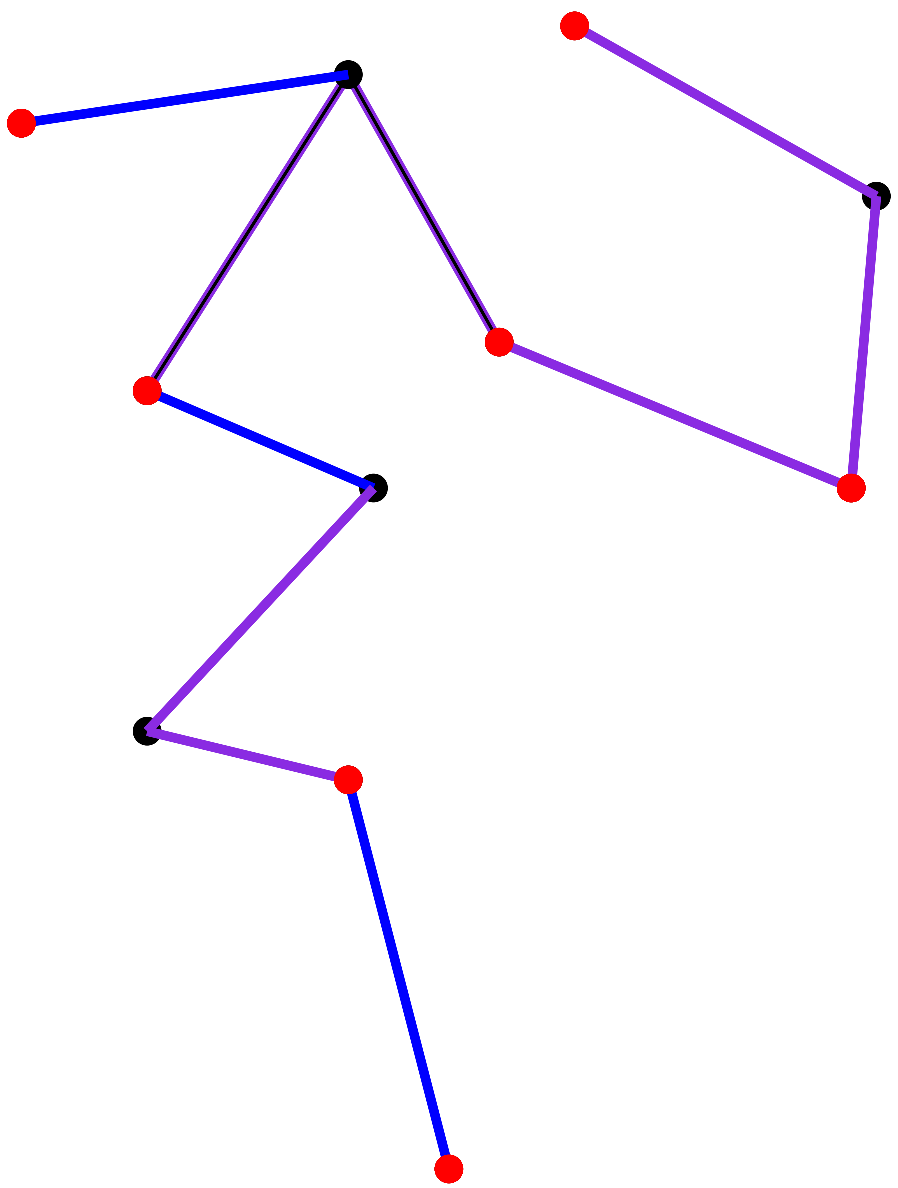
Exact solution T

Uncontract F

Steiner Tree in Planar Graphs

(Borradaile-Klein-Mathieu 2009)



Find a special subgraph spanning the terminals

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Reduce to bounded treewidth

Light set of edges F: Length$(F) \leq \epsilon$OPT

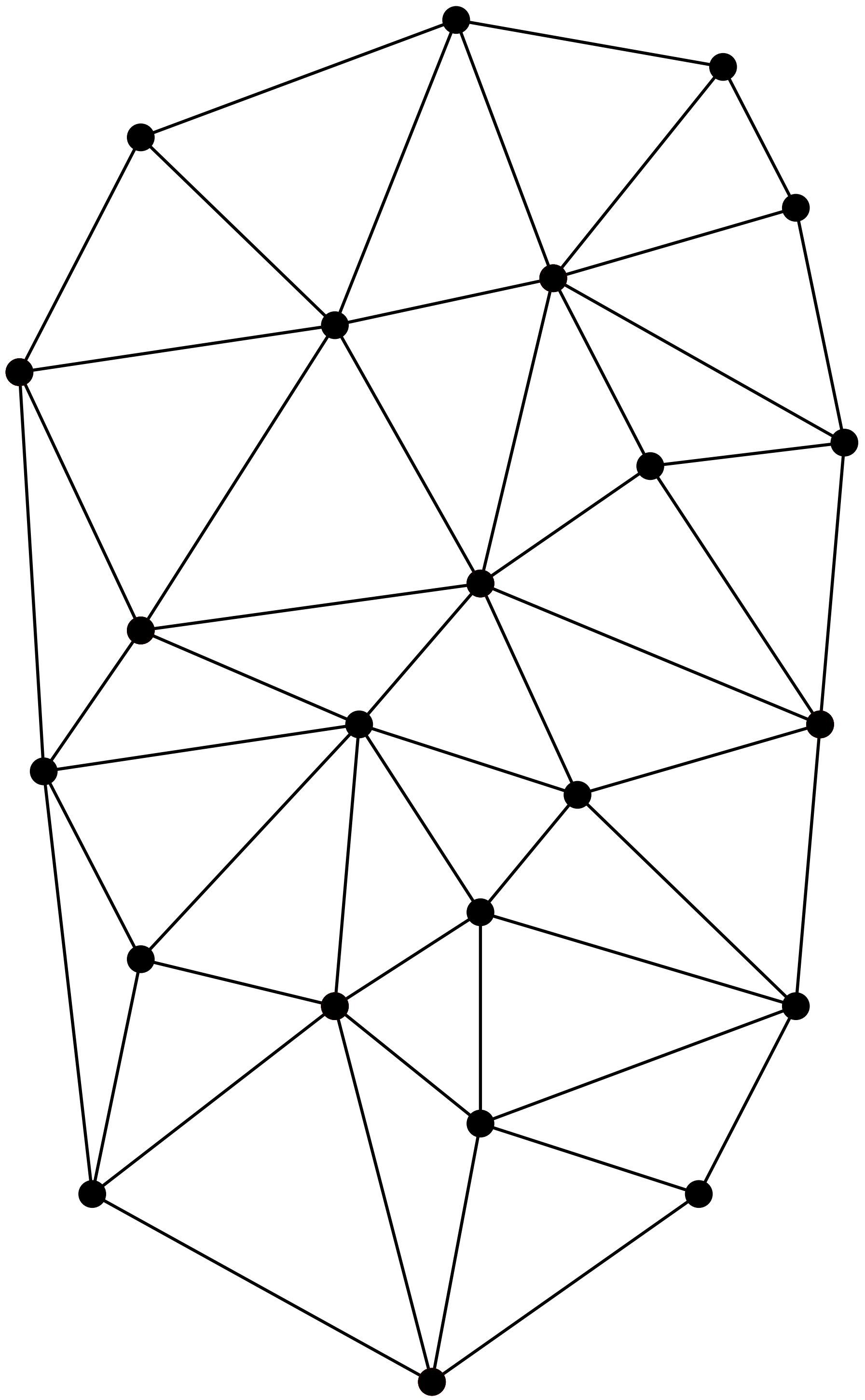
G/F has bounded treewidth

Exact solution T

Uncontract F

Length(T) + Length$(F) \leq (1+\epsilon)$OPT

Capturing Important Vertices



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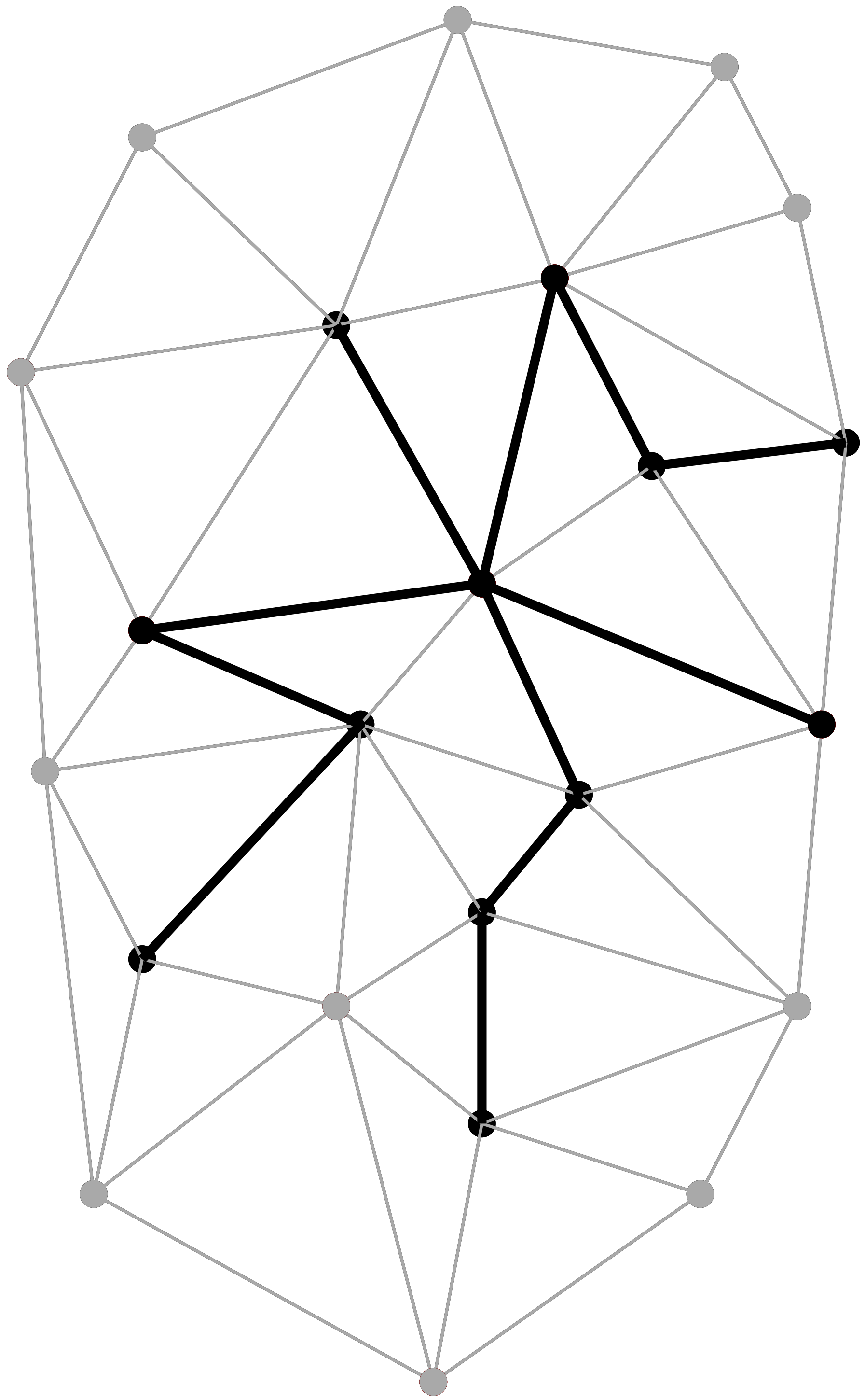

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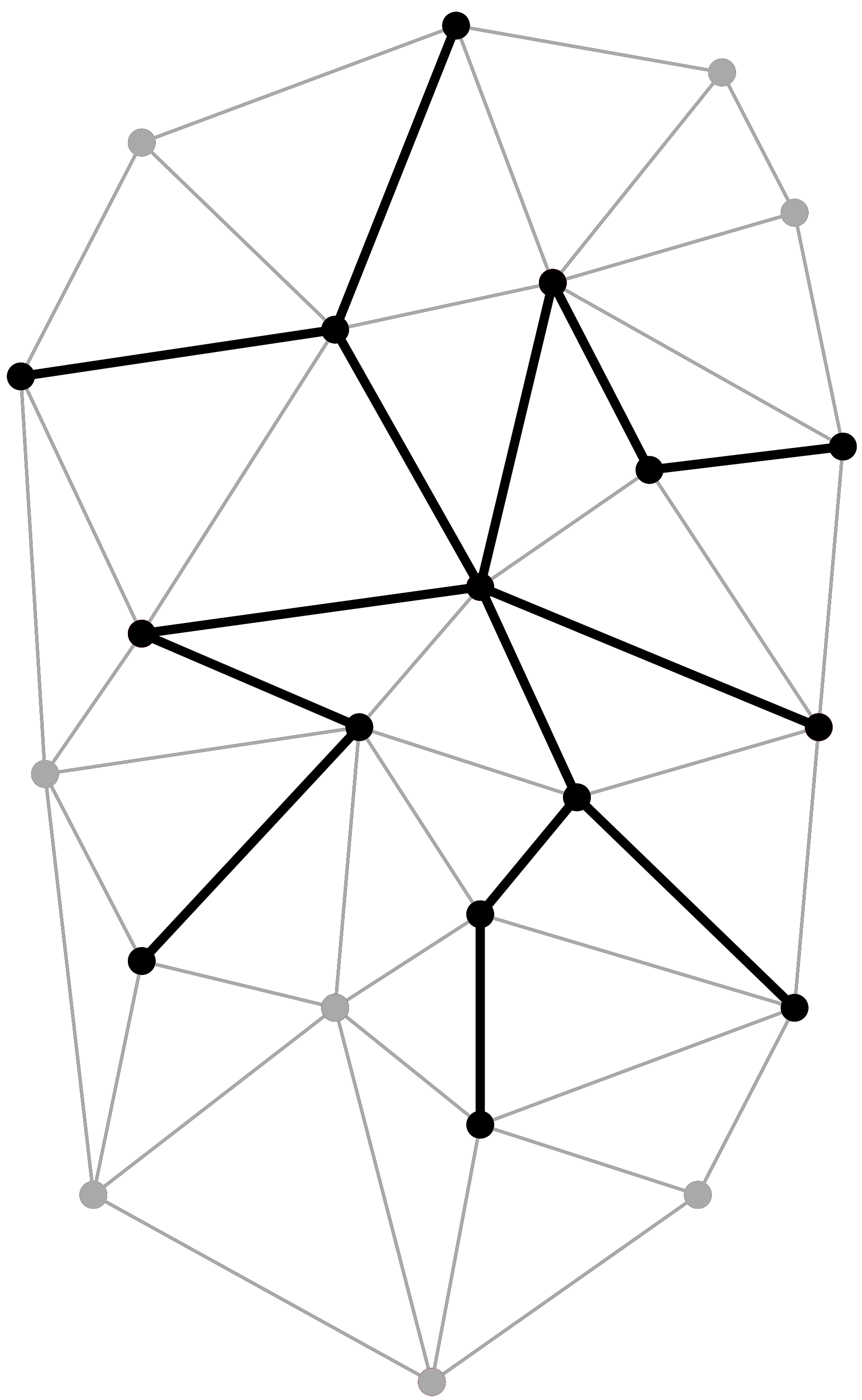
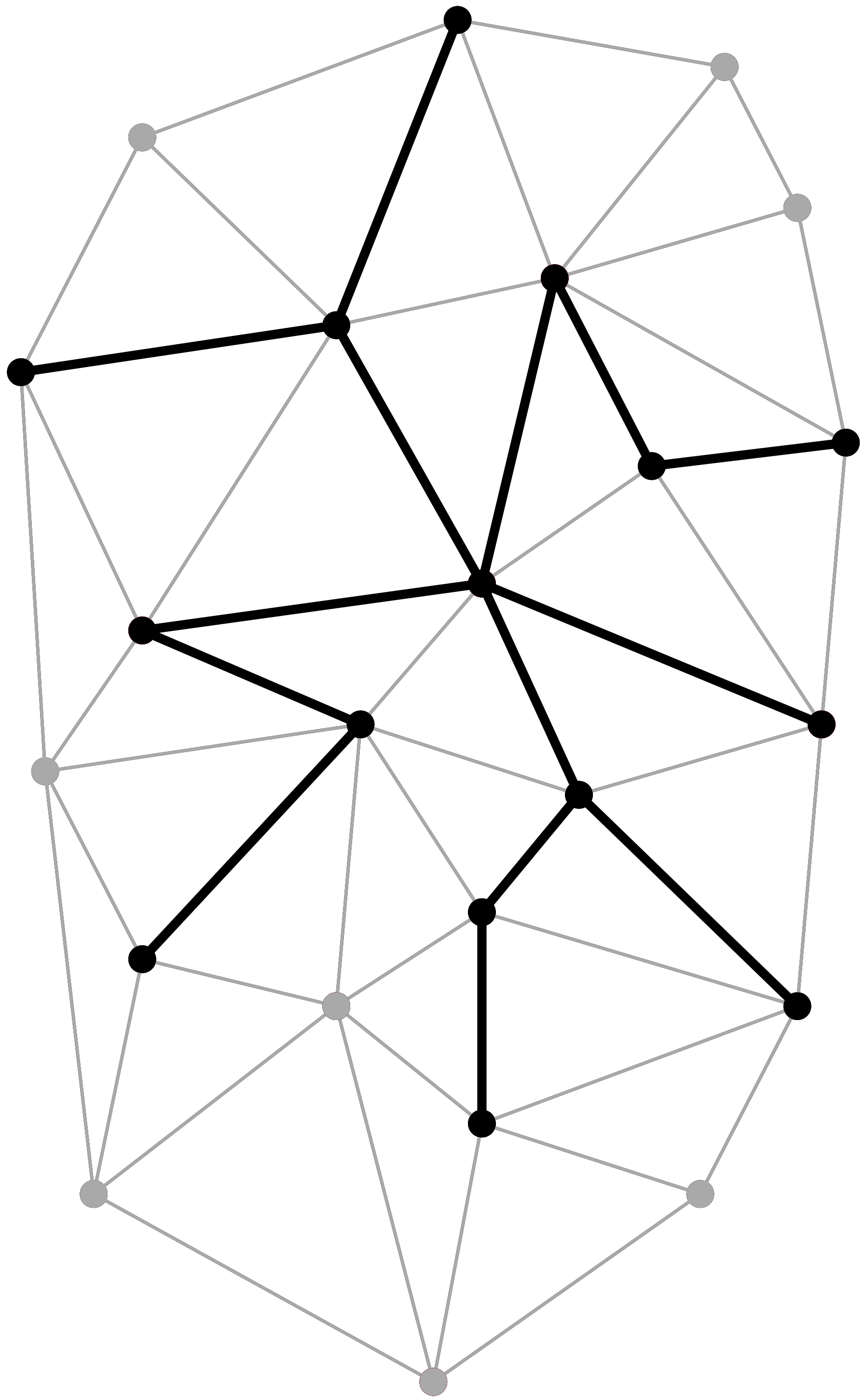


Increase penalties

Capturing Important Vertices

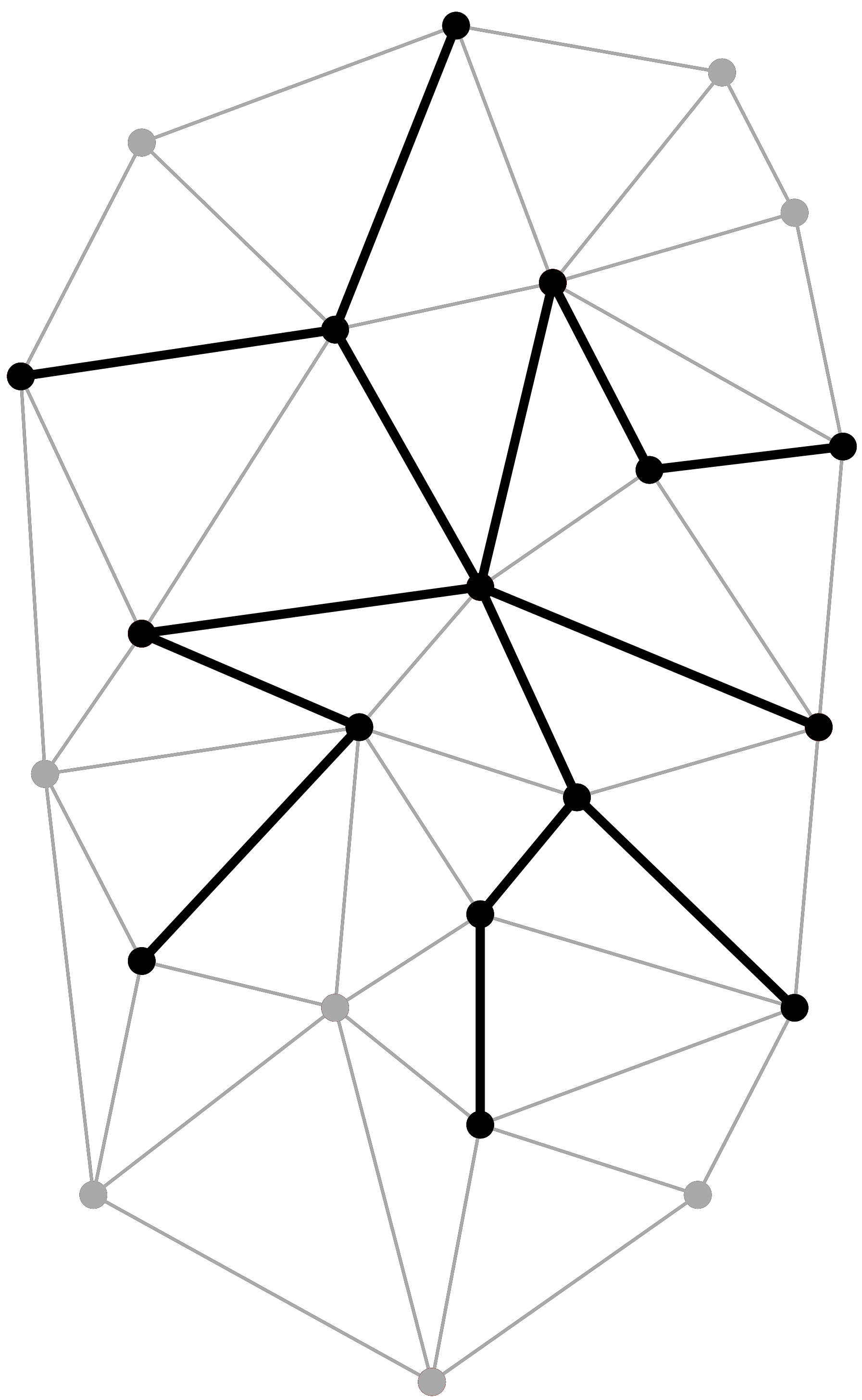


Increase penalties

$$\pi'_v = \frac{1}{\epsilon} \cdot \pi_v$$

Run Goemans-Williamson algorithm

Capturing Important Vertices



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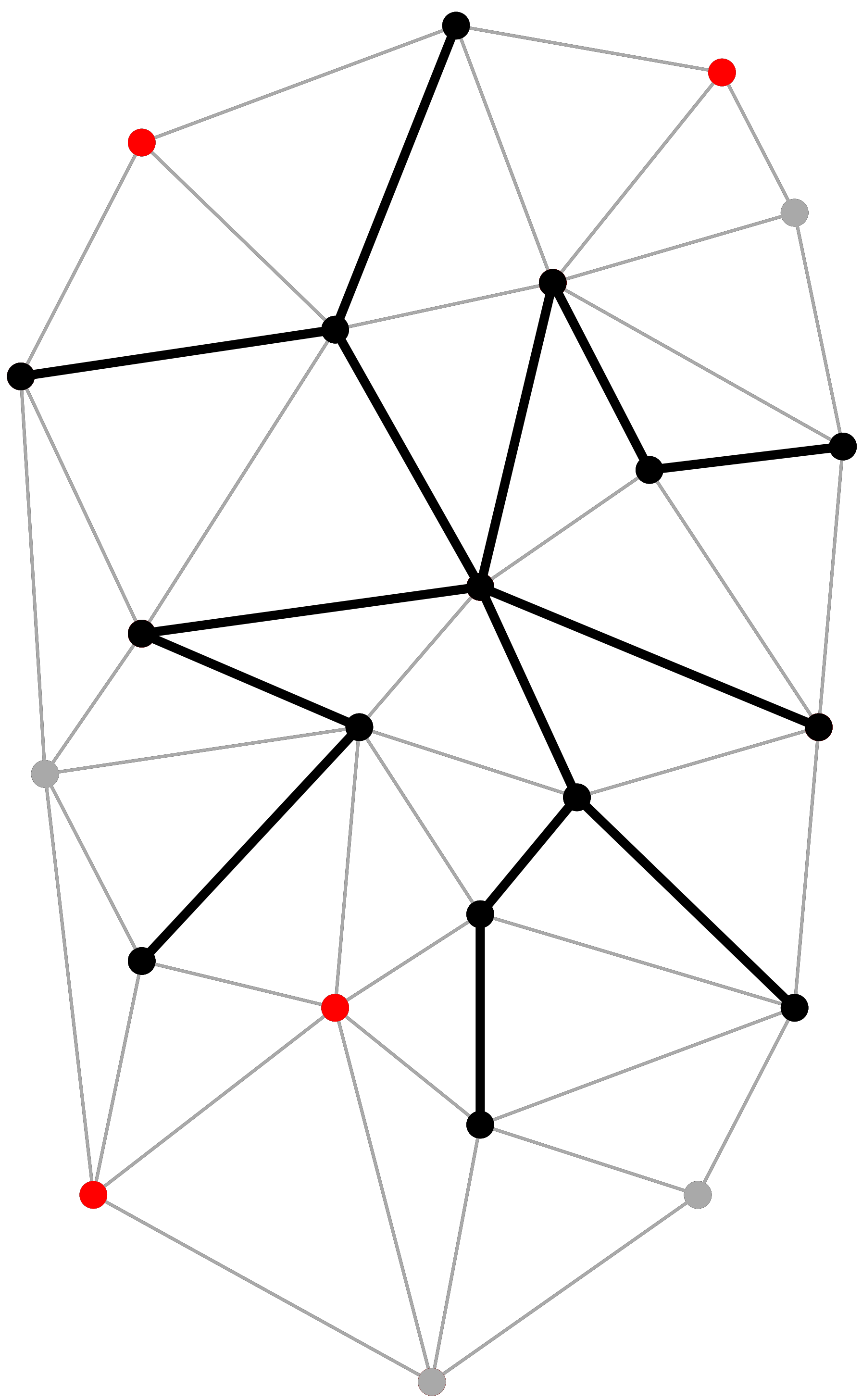
$\text{Length}(T) + \pi'(V - T) \leq 2\text{OPT}' \leq \frac{2}{\epsilon}\text{OPT}$

$\pi'(T^* - T) \leq \text{OPT}$

Capturing Important Vertices



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Dual fitting wrt Steiner Tree dual LP

Prize Collecting Steiner Forest

Primal-dual algorithm

Scale penalties to capture important terminal pairs

Adapt Steiner Forest spanner construction

APX-hard on series parallel graphs (Bateni-Hajiaghayi-Marx 2010)

Remarks and Open Problems

Reductions extend to bounded genus graphs

Open problems

- PTAS for planar k-MST?
- PTAS for planar Orienteering?

Thank You

Questions?