

CS257: Lecture 21 "Quiz"

1. How many additional constraints need to be set for a general cubic spline?

2. Three ways of imposing the conditions are:

(a) Natural Cubic Splines: require _____

(b) Not-A-Know Cubic Splines: require _____

(c) Fixed-end-slope Cubic Splines: require _____

3. Do there exist values a , b , c , and d so that the function

$$S(x) = \begin{cases} ax^3 + x^2 + cx & -1 \leq x \leq 0 \\ bx^3 + x^2 + dx & 0 \leq x \leq 1 \end{cases}$$

is a natural cubic spline function that agrees with the absolute value function $f(x) = |x|$ at the knots $-1, 0, 1$?

4. How many additional conditions are needed to uniquely specify a spline of degree 4 over n knots?

5. Some interesting links:

- <http://www.tinaja.com/cubic01.asp>
- <http://www.cse.unsw.edu.au/lambert/splines/natcubic.html>
- <http://www.math.ucla.edu/baker/java/hofer/Spline.htm>
- <http://ibiblio.org/e-notes/Splines/Intro.htm>

Listing 1: natsplinecoeff

```
1 Input:  $1 \times (n+1)$  arrays  $x$  and  $y$ 
2 Output:  $1 \times (n+1)$  array  $z$ 
3
4 Initialize:  $1 \times n$  arrays  $h, b, u, v$ 
5
6 for  $i = 1$  to  $n$ 
7    $h_i \leftarrow x_{i+1} - x_i$ 
8    $b_i \leftarrow (y_{i+1} - y_i)/h_i$ 
9 end
10  $u_2 \leftarrow 2(h_1 + h_2)$ 
11  $v_2 \leftarrow 6(b_2 - b_1)$ 
12 for  $i = 3$  to  $n$ 
13    $u_i \leftarrow 2(h_i + h_{i-1}) - h_{i-1}^2/u_{i-1}$ 
14    $v_i \leftarrow 6(b_i - b_{i-1}) - h_{i-1}v_{i-1}/u_{i-1}$ 
15 end
16  $z_{n+1} \leftarrow 0$ 
17 for  $i = n$  to  $1$  step  $-1$ 
18    $z_i \leftarrow (v_i - h_i z_{i+1})/u_i$ 
19 end
20  $z_0 \leftarrow 0$ 
```

Listing 2: splineeval

```
1 Input:  $1 \times (n+1)$  arrays  $x, y, z$ 
2 Input: evaluation value  $\hat{x}$ 
3
4 Output:  $S(\hat{x})$ 
5
6 for  $i = n$  to  $1$  step  $-1$ 
7   if  $\hat{x} - x_i \geq 0$  then exit loop
8 end
9
10  $h \leftarrow x_{i+1} - x_i$ 
11  $\alpha \leftarrow (z_i/2) + (x - x_i)(z_{i+1} - z_i)/(6h)$ 
12  $\alpha \leftarrow -(h/6)(z_{i+1} + 2z_i) + (y_{i+1} - y_i)/h + (x - x_i)\alpha$ 
13  $\alpha \leftarrow y_i + (x - x_i)\alpha$ 
14 return  $\alpha$ 
```