Lecture 1

Numerical Methods

L. Olson

Department of Computer Science
University of Illinois at Urbana-Champaign

January 17, 2006

Numerical Methods?

Numerical Analysis?

Numerical Calculation vs. Symbolic Calculation

- Numerical Calculation: involve numbers directly
  - manipulate numbers to produce a **numerical** result
- Symbolic Calculation: symbols reprepsent numbers
  - manipulate symbols according to mathematical rules to produce a symbolic result

Example (numerical)

$$\frac{(17.36)^2 - 1}{17.36 + 1} = 16.36$$

Example (symbolic)

$$\frac{x^2 - 1}{x + 1} = x - 1$$

Analytic Solution vs. Numerical Solution

- Analytic Solution (a.k.a. symbolic): The exact numerical or symbolic representation of the solution
  - may use special characters such as π, e, or $\tan(83)$
- Numerical Solution: The computational representation of the solution
  - entirely numerical

Example (analytic)

$$\frac{1}{4}$$

$$\frac{1}{3}$$

$$\pi$$

$$\tan(83)$$

Example (numerical)

$$0.25$$

$$0.33333\ldots(?)$$

$$3.14159\ldots(?)$$

$$0.88472\ldots(?)$$

Numerical Computation and Approximation

- Numerical Approximation is needed to carry out the steps in the numerical calculation. The overall process is a numerical computation.

Example (symbolic computation, numerical solution)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} - 1 = \frac{1}{12} = 0.083333333\ldots$$

Example (numerical computation, numerical approximation)

$$0.500 + 0.333 + 0.250 - 1.000 = 0.083$$

Method vs. Algorithm vs Implementation

- Method: a general description of a process
- Algorithm: a detailed description of the method
- Implementation: a particular instantiation of the the algorithm

Example

Goal: Make a pot of coffee

Method

Use drip coffeemaker (versus percolator or espresso machine)

Method vs. Algorithm vs Implementation

- Method: a mathematical description of a calculation
- Algorithm: a detailed sequence of steps
- Implementation: a particular instantiation of the the algorithm

Algorithm 1

1. Grind fresh beans
2. Place new filter in filter holder
3. Measure 4 tablespoons into the filter
4. Wash and rinse the coffeepot
5. Fill coffeemaker with 4 cups of water
6. Start the machine

Algorithm 2

1. Take coffee from the can
2. Place new filter in filter holder
3. Pour coffee into filter
4. Empty coffeepot
5. Fill coffeemaker with water
6. Start the machine

- accuracy is much different
- results are not consistent after each run
- BUT algorithm 2 is cheaper (both in speed and in resources)

Method vs. Algorithm vs Implementation

- Method: a general description of a process
- Algorithm: a detailed description of the method
- Implementation: a particular instantiation of the the algorithm

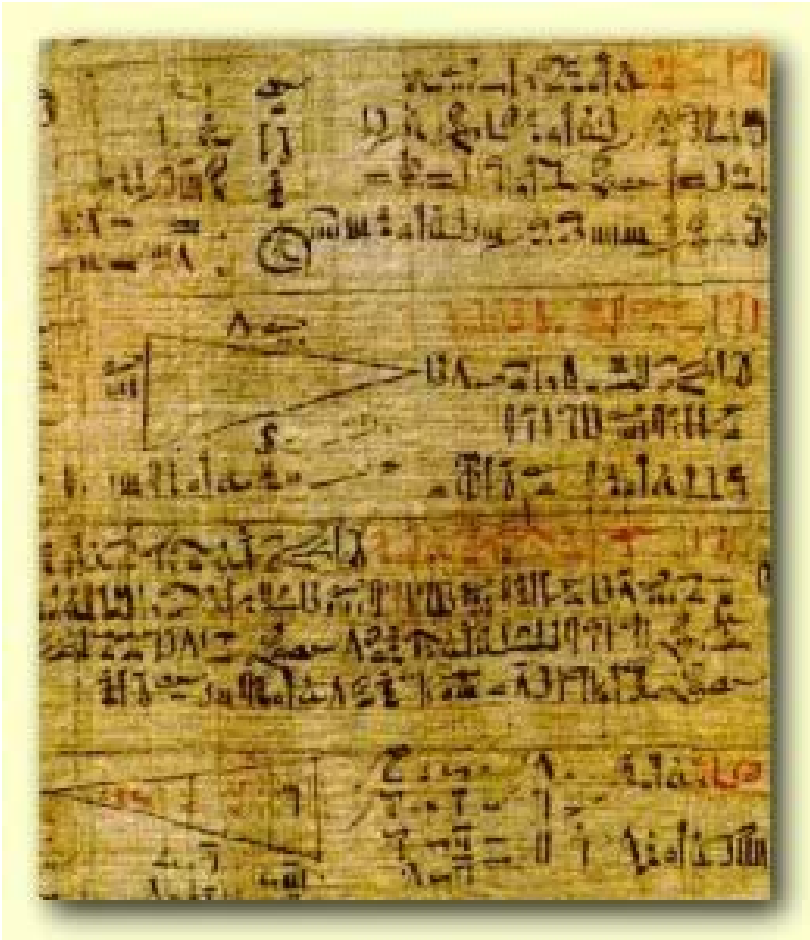
Implementation 1

1. Grind fresh Starbucks beans in a Williams and Sonoma 1000 grinder
2. Place new unbleached Mr. Coffee filter in filter holder
3. Measure 4 tablespoons with hand into the filter
4. Wash and rinse the coffeepot
5. Fill KitchenAid coffeemaker with 4 cups water
6. Start the machine

- expensive implementation
- is it worth it?

Algorithms

- date to 1650 BC: The Rhind Papyrus of ancient Egypt contains 85 problems; many use numerical algorithms (T. Chartier, Davidson)



Archimedes

- 287-212BC developed the "Method of Exhaustion"
- Method for determining π
  - find the length of the permieter of a polygon inscribed inside a circle of radius $1/2$
  - find the permiter of a polygon circumscribed outside a circle of radius $1/2$
  - the value of π is between these two lengths

Method of Exhaustion

- A circle is not a polygon
- A circle **is** a polygon with an infinite number of sides
- C_n = circumference of an n-sided polygon inscribed in a circle of radius $1/2$
- $\lim_{n\to\infty} = \pi$
- Archimedes deterimined

$$\frac{223}{71} < \pi < \frac{22}{7}$$
$$3.1408 < \pi < 3.1429$$

- two places of accuracy....
- `see http://www.pbs.org/wgbh/nova/archimedes/pi.html`

Method of Machin

- Around 1700, John Machin discovered the trig identity

$$\pi = 16 \arctan\left(\frac{1}{5}\right) - 4 \arctan\left(\frac{1}{239}\right)$$

- Led to calculation of the first 100 digits of π
- Uses the Taylor series of arctan in the algorithm

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \ldots$$

- Used until 1973 to find the first Million digits

Numerical Analysis

Definition (Trefethen)

Study of algorithms for the problems of continuous mathematics

We've been doing this since Calculus (and before!)

- Riemann sum for calculating a definite integral
- Newton's Method
- Taylor's Series expansion + truncation

Big Questions

- How algorithms work and how they fail
- Why algorithms work and why they fail

- Connects mathematics and computer science
- Need mathematical theory, computer programming, and scientific inquiry

Numerical Analysis

A Numerical Analyst needs

- computational knowledge (e.g. programming skills)
- understanding of the application (physical intuition for validation)
- mathematical ability to construct and meaningful algorithm

Numerical Analysis

Numerical focus:

Approximation An approximation solution is sought. How close is this to the desired solution?

Efficiency How fast and cheap (memory) can we compute a solution?

Stability Is the solution sensitive to small variations in the problem setup?

Error What is the role of finite precition of our computers?

Numerical Analysis

Why?

- Numerical methods improve scientific simulation
- Some disasters attributable to bad numerical computing (Douglas Arnold)
  - The Patriot Missile failure, in Dharan, Saudi Arabia, on February 25, 1991 which resulted in 28 deaths, is ultimately attributable to poor handling of rounding errors.
  - The explosion of the Ariane 5 rocket just after lift-off on its maiden voyage off French Guiana, on June 4, 1996, was ultimately the consequence of a simple overflow.
  - The sinking of the Sleipner A offshore platform in Gandsfjorden near Stavanger, Norway, on August 23, 1991, resulted in a loss of nearly one billion dollars. It was found to be the result of inaccurate finite element analysis.

I thought we were studying "Numerical Methods"?!

Numerical analysis is the study of numerical methods

- Numerical Analysis: understanding general behavior of numerical methods
- Numerical Methods: understanding **how** to apply certain methods to solve specific tasks
- As programmers, we need to understand the concepts of numerical analysis and implementation aspects of the numerical method

We thus focus on

- Matlab implementation
  - fast learning curve
  - quick time-to-production: low development times
  - a major develop environment in scientific computing
  - integrated graphics
- Errors in computation
- specific methods: root finding, integrating, interpolation, etc.

look at areas of application along the way:

Engineering simulating physical systems from industry or the environment, e.g. design/ analysis of buildings/bridges/aircraft/automobiles, weather prediction, prediction of traffic flow.

Biology Simulation of biological processes, e.g. spread of disease, population growth, blood flow/circulation/nerve firing. (Computational genetics often uses discrete algorithms, drawing from combinatorics, rather than numerical analysis.)

Computer graphics image compression, representation of curves/surfaces/lighting

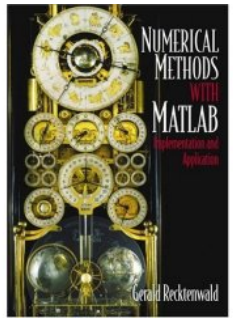
Economics/Finance modeling/simulation of financial data. Scientific computing: design of algorithms for high performance/parallel computing.

Social/behavioral sciences simulation of social processes.

The Boring Part

`http://www.cs.uiuc.edu/class/sp06/cs257/`

- Book: Numerical Methods With Matlab: Implementation and Application by Gerald Recktenwald
- UIUC bookstore: $102 new, $77 used
- Amazon: $96 new, $48 used
- eCampus: $99 new



Luke's contact:

- Office: 4312 Siebel Center
- Office Hours: TR 11-12
- email: lukeo AT uiuc.edu
- IM: cs256prof

Adam Reichert, Reza Zamani Nasab, Jacob Schroder contact:

- Office: 0228 Siebel Center
- Office Phone: 265-9212
- Office Hours: TBD
- emails: {areiche2, zamani, jschrod3} AT uiuc.edu

The Painful Part

`http://www.cs.uiuc.edu/class/sp06/cs257/`

	Homework	25%
	Exam 1	20%
Grades:	Exam 2	20%
	Final Exam	25%
	Project	10%

Homework:

- weekly
- due at 4pm at the TAs office (box)
- submitted in paper
- do not email
- source code included upon request
- no graphics by hand
- accepted for 50% credit up to one week late
- no dropped scores
- collaborate!
  - cite collaborators at the top
  - hand in your **own** work in your **own** words

The Interesting Part

`http://www.cs.uiuc.edu/class/sp06/cs257/`

Project:

- "peer" or "collaborative" learning project
- goal: build a "how-to" or "guide" to complete or explain a numerical method or task
- wiki based: `http://walleye.cs.uiuc.edu/wiki/`
- grade based on content, style, collaboration, progress
- examples:
  - Calculating the Google Page Rank (Luke has started)
  - Determining π with Machin's Method and Archimedes Method (open)
  - Numerical Methods and Fractals (open)
  - Errors in the Pentium Processor (open)
  - etc.
- Logins will be available shortly
- Jacob is the primary TA for the project

Finally...

`http://www.cs.uiuc.edu/class/sp06/cs257/`

Schedule and Notes:

- on the web
- everything is **tentative** including midterm exams
- final exam is fixed

Questions?