CS 257: Numerical Methods
Spring 2006

Homework, Set 9 **Due Thursday April 13, 2006**

(-) Start EACH PROBLEM on a SEPARATE piece of paper (This is important since we may assign each problem to a different grader).

(-) Put your NETID and HW NUMBER on top of EACH PAGE clearly, e.g. "netid: zamani hw2".

(-) Write descriptive solutions. Comment your code!

(-) Include your curves/graphs (and other supporting materials) in your write-up.

(-) Don't use handwritten code (unless you want to lose points), copy-paste your code into your write-up or attach a proper print of code to your papers.

(-) Please write everything in a "portrait" style (not landscape).

(-) Please number problems according to numbers presented in the homework write-up that appears on the course page, NOT according to the numbers in the textbook.

(-) Please type your homework or hand-write it legibly (but yet attach a print of your codes to your handwritten stuff).

(-) Show that your code works (even if the problem doesn't explicitly asks to test your code!)

Goals of this homework:

- Trapezoid and Simpson integration rules by hand
- Test the integration rules with m-scripts

Throughout the homework, pay particular attention to

- The number of function evaluations needed in the integration rules (Trapezoid and Simpson)
- The accuracy of each method

You will need these two m-scripts:

http://www.cs.uiuc.edu/class/sp06/cs257/homework/simpson.m

http://www.cs.uiuc.edu/class/sp06/cs257/homework/trapezoid.m

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1. Trapezoid (by hand) Use the composite trapezoid rule at the points 1, $\frac{4}{3}$, and 2 to approximate $\int_1^2 \frac{1}{x}\,dx$.

2. Trapezoid Compute an approximation to $\int_0^1 \frac{1}{x^2+1}\,dx$ using the composite trapezoid rule with 3 points and with 5 points. Compare each with the exact value of the integral.

3. Trapezoid How large must n be if the composite trapezoid rule is to estimate $\int_0^\pi \sin(x)\,dx$ with an error less than or equal to 10^{-12}.

4. Trapezoid We have data

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
$f(x)$	10	8	7	6	5

and want to approximate $\int_1^2 f(x)\,dx$. Compute and estimate using the composite trapezoid rule.

5. Simpson Compute an approximation to $\int_0^1 \frac{1}{x^2+1}\,dx$ using the composite simpson rule with 3 points. Compare with the exact value of the integral.

6. Simpson A function f has the values

x	1	1.25	1.5	1.75	2
$f(x)$	10	8	7	6	5

6.1 Use the Simpson's Rule at $x = 1$, 1.5, 2 to approximate $\int_1^2 f(x)\,dx$.

6.2 Use the composite Simpson's Rule at $x = 1$, 1.25, 1.5, 1.75, 2 to approximate $\int_1^2 f(x)\,dx$.

7. Trapezoid/Simpson Run the MATLAB scripts for trapezoid and simpson to compute approximate of the integral $\int_0^{2\pi} \cos(2x)e^{-x}\,dx$. Fill in the following table

n	$\hat{I}_T$	$error_T$	$\hat{I}_S$	$error_S$
1				
2				
3				
4				
5				
6				
7				
8				

$\hat{I}_T$ and $\hat{I}_S$ are approximations to the integral using trapezoid and simpson respectively.

8. Trapezoid/Simpson Fill in the following table:

	error	function evaluations	h
basic Trapezoid			
composite Trapezoid	$\mathcal{O}(h^2)$	$2n$	$x_k - x_{k-1}$
basic Simpson	$\mathcal{O}(h^5)$	3	$b - a$
composite Simpson			