CS 257: Numerical Methods
Spring 2006

Homework, Set 7 **Due Thursday March 30, 2006**

- **(-)** Start EACH PROBLEM on a SEPARATE piece of paper (This is important since we may assign each problem to a different grader).
- **(-)** Put your NETID and HW NUMBER on top of EACH PAGE clearly, e.g. "netid: zamani hw2".
- **(-)** Write descriptive solutions. Comment your code!
- **(-)** Include your curves/graphs (and other supporting materials) in your write-up.
- **(-)** Don't use handwritten code (unless you want to lose points), copy-paste your code into your write-up or attach a proper print of code to your papers.
- **(-)** Please write everything in a "portrait" style (not landscape).
- **(-)** Please number problems according to numbers presented in the homework write-up that appears on the course page, NOT according to the numbers in the textbook.
- **(-)** Please type your homework or hand-write it legibly (but yet attach a print of your codes to your handwritten stuff).
- **(-)** Show that your code works (even if the problem doesn't explicitly asks to test your code!)

We will study three types of interpolation in this homework:

1. using Monomial basis

2. using Lagrange interpolating polynomials

3. using Newton interpolation

Through this homework, we will highlight particular aspects of each of these interpolation strategies:

1. Monomials: poor conditioning, propensity for round-off

2. Lagrange: use of existence/uniqueness, Kronecker Delta function, wiggles

3. Newton: divided differences, connection to nested form

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(1) Monomials Given a 4×4 diagonal matrix A with entries 5, 4, 100, and 200, what is the condition number (use the 1-norm)? What is the condition number of of A^{-1}? (compute everything by hand)

(2) Monomials Given the following data

x	5000	5005	5010	5015	5020	5025
y	1	1	1	-1	-1	-1

determine coefficients for the minimum degree polynomial that interpolates this data using a Monomial basis. Plot your polynomial against this data. Also determine with Matlab, κ for the associated Vandermonde matrix. *note: attach your Matlab script. It should only be a few lines.*

(3) Lagrange By hand, use the Lagrange interpolation process to obtain a polynomial of least degree that interpolates the following data:

x	0	2	3	4
y	7	11	28	63

(4) Lagrange For the four interpolation nodes $-1, 1, 3, 4$, what are the Lagrange basis functions? State them explicitly and plot them graphically.

(5) Lagrange Given the function

$$f(x) = (1 + x^2)^{-1}$$

Plot the interpolatin polynomial of 10th degree using Lagrange interpolation for each of the following sets of data points:

- equi-spaced: $x_i = -5, -4, -3, \ldots, 3, 4, 5$
- non-equi-spaced: $x_i = \cos\left[\frac{2i+2}{2n+2}\pi\right]$

(6) Polynomials The polynomials

$$p(x) = 5x^3 - 27x^2 + 45x - 21$$
$$q(x) = x^4 - 5x^3 + 8x^2 - 5x + 3$$

both interpolate the data

x	1	2	3	4
y	2	1	6	47

Why is this not a violation of the uniqueness part of the theorem on existence of polynomial interpolation?

(7) Newton Find the polynomial p of least degree that takes the values: $p(0) = 2$, $p(2) = -4$, $p(3) = -4$, and $p(5) = 82$. Use divided differences to get the correct polynomial. It is not necessary to completely simplify your expression.

(8) Newton Given the data

x	0	1	2	4	6
$f(x)$	1	9	23	93	259

First construct the divided-difference table. Then using Newton's interpolating polynomial, find an approximation to $f(4.2)$. *Hint: start with 9 and use terms of the form* $(x - 1)$.

(9) Newton Construct Newton's interpolating polynomial for the data shown

x	0	2	4	6
$f(x)$	7	11	28	63

Without simplifying it, write the polynomial obtained in nested form for easy evaluation.

(10) Newton Develop the divided-difference table from the following data. Write down the interpolating polynomial and re-arrange it for fast computation without simplifying.

x	0	1	3	2	5
y	2	1	5	6	-183