CS 257: Numerical Methods
Spring 2006

Homework, Set 4 **Due Thursday February 16, 2006**

(-) Start EACH PROBLEM on a SEPARATE piece of paper (This is important since we may assign each problem to a different grader).

(-) Put your NETID and HW NUMBER on top of EACH PAGE clearly, e.g. "netid: zamani hw2".

(-) Write descriptive solutions. Comment your code!

(-) Include your curves/graphs (and other supporting materials) in your write-up.

(-) Don't use handwritten code (unless you want to lose points), copy-paste your code into your write-up or attach a proper print of code to your papers.

(-) Please write everything in a "portrait" style (not landscape).

(-) Please number problems according to numbers presented in the homework write-up that appears on the course page, NOT according to the numbers in the textbook.

(-) Please type your homework or hand-write it legibly (but yet attach a print of your codes to your handwritten stuff).

(-) Show that your code works (even if the problem doesn't explicitly asks to test your code!)

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(1) (NMM Ch. 6 #3) Apply the `brackPlot` function to each of the following equations. How many roots are in the designated interval?

(a) $\cos(x) = x$ for $-2\pi \leq x \leq 2\pi$.

(b) $\cos(5x) = x$ for $-2\pi \leq x \leq 2\pi$.

(c) $\tan(x) = 0$ for $-2\pi \leq x \leq 2\pi$.

(d) $\sin(1/x) = 0$ for $\pi/50 \leq x \leq \pi/3$.

(2) Write a bisection function that has

- input: `func`, string to a function func.m
- input: `maxit`, maximum iterations allowed
- input: `tol`, relative error tolerance
- input: `a`, `b`, initial bracket
- output: `n`, the number of iterations
- output: `err`, final relative error bound
- output: `x`, final approximation
- make all inputs optional

(A) Plot $y = x$ and $y = sin(x)$.

(B) Use your bisection function to find an approximate to within 10^{-3} to the first positive value of x with $x = 2\sin(x)$.

(C) Show your convergence history: at each step display iterate k, approxmate root x_k, relative error err_k (eq. 6.9), error ratio $\frac{err_{k-1}}{err_k}$.

(3) Write a fixed point function that has

- input: func, string to a function func.m
- input: x0, initial guess of the root
- input: maxit, maximum iterations allowed
- input: tol, relative approximate error tolerance
- output: n, the number of iterations
- output: err, final relative approximate error bound
- output: x, final approximation
- make all inputs optional

(A) (NMM Ch.6 #17) Compute the equilibrium composition of a mixture of carbon monoxide and oxygen gas at one atmosphere. Determining the final composition requires solving

$$3.06 = \frac{(1-x)(3+x)^{1/2}}{x(1+x)^{1/2}}$$

for x. Obtain a fixed point iteration formula for finding the roots of this equation. Implement your formula in a MATLAB function and use your function to find x. If your formula does not converge, develop one that does.

(B) Report the convergence history: iterate k, approximate root x_k, and the relative approximate error $\frac{x_k - x_{k-1}}{x_k}$.

(4) Write a Newton Method function that has

- input: func, string to a function func.m
- input: x0, initial guess of the root
- input: maxit, maximum iterations allowed
- input: xtol, relative error tolerance for x
- input: ftol, relative error tolerance for f
- output: n, the number of iterations
- output: err, final relative approximate error bound
- output: x, final approximation
- make all inputs optional
- *see Listing 6.6*

(A) Use your Newton method function to determine the lowest positive root of $f(x) = 6\sin(x)e^x - 1$. Use $x_0 = 0.3$. Choose a reasonable $xtol$ and $ftol$. Report iterates, errors (both), and the approximate root at each step.

(B) Use your Newton method function to determine a real root for $f(x) = -2 + 6x - 4x^2 + 0.5x^3$. Use an initial guess of (i) $x_0 = 4.5$ and (ii) $x_0 = 4.43$. Discuss and use graphical and analytical methods to explain any peculiarities in your results.

(C) Use your Newton method function to determine the positive root of $f(x) = x^{10} - 1$. Use $x_0 = 0.5$. Discuss and use graphical and analytical methods to explain any peculiarities in your results. [*hint: look at the convergence* **rate**].

DO NOT write up problems below this line. They are just pointers to good problems for practice.

(I) (NMM Ch.6 #1)

(II) (NMM Ch.6 #23)

(III) (NMM Ch.6 #26)