CS 257: Numerical Methods
Spring 2006

Homework, Set 1 **Due Thursday January 26, 2006**

Write descriptive solutions. Comment your code!

(1) (NMM Ch. 2 #5) Use the linspace function to create vectors identical to those obtained with the statements that follow. Use multiple statements where necessary. (Use MATLAB's built-in norm function to test whether two vectors are equal *without* printing the elements.)

(a) x = 0 : 10

(b) x = 0 : 0.2 : 10

(c) x = -12 : 12

(d) x = 10 : -1 : 1

(2) (NMM Ch. 2 #14) Use the diag function to create the symmetric, $n \times n$, tridiagonal matrix

$$D = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

(*Hint:* The two parameter form of diag will be helpful.)

(3) (NMM Ch. 2 #16) Write a one-line expression to create the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ -1 & -1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & -1 & 1 \end{bmatrix}$$

(*Hint:* One solution uses matrix addition and the built-in tril, ones, eye, and zeros commands.)

(4) (NMM Ch. 2 #29) A theoretical model of an athlete in a sprint race is (see W.G. Pritchard, *Mathematical Models of Running,* SIAM Review, Vol. 35, No. 3, pp. 359-379, September 1993)

$$x(t) = a\tau[t - \tau(1 - e^{-t/\tau})]$$

where x is distance traveled in time t, and a and τ are constants for a given runner and distance of the race. Measured values of $x(t)$ for Carl Lewis and Ben Johnson in 100-meter final at the 1987 Championship in Rome are given in the following table:

$x(m)$	0	10	20	30	40	50	60	70	80	90	100
Lewis time(s)	0	1.94	2.96	3.91	4.78	5.64	6.50	7.36	8.22	9.07	9.93
Johnson time(s)	0	1.84	2.86	3.80	4.67	5.53	6.38	7.23	8.10	8.96	9.83

A curve fit to Lewis data gives $\tau = 0.739\ s$ and $a = 14.4\ m/s^2$. Plot the measured times for Lewis and Johnson versus the theoretical model of Carl Lewis on the same graph. Use a solid line for theoretical model, and use symbols with no line for the measured data. To make the theoretical curve look smooth at small times, choose a small time increment, say, 0.2 s. Label the axes and add a legend to the plot.

Get file sprint.dat from here

http://www.cs.uiuc.edu/class/sp06/cs257/homework/sprint.dat

(5) (NMM Ch. 2 #33) Surface plots do not need to be plot on a rectangular grid. Create a surface plot of $z = 2 + x^2 + y^2$ defined on the domain bounded by the circle $x^2 + y^2 = 5$. The trick is to evaluate the function at points inside the circle. MATLAB's plotting routines will do the hard work. First define the r and θ vectors with

```
>> r = linspace(0,sqrt(5),20);
>> theta = linspace(0,2*pi,10)
```

Then evaluate the X and Y *matrices* with outer products

$$X_{i,j} = r_i \cos(\Theta_j) \quad \text{and} \quad Y_{i,j} = r_i \sin(\Theta_j)$$

The surface function $Z = f(X, Y)$ is evaluated just as in Example 2.7. Contrast the resulting plot from that obtained when X and Y result from

```
>> x = linspace(-5,5,20);
>> [X,Y] = meshgrid(x,x);
```

(6) Show that $\sum_{n=0}^{\infty} \frac{1}{(2n+1)(2n+2)}$ converges to $\log(2)$ (the *natural* log). Do not use loops.