AM 194: Iterative Methods

Brown University **Fall 2005**
Homework, Set 3 **Due Wed. April 11, 2005**

Precision Consider the 1-D beam problem:

$$A = \begin{bmatrix}
5 & -4 & 1 & & & & & & & & & \\
-4 & 6 & -4 & 1 & & & & & & & 0 & \\
1 & -4 & 6 & -4 & 1 & & & & & & & \\
 & 1 & -4 & 6 & -4 & 1 & & & & & & \\
 & & \ddots & \ddots & \ddots & \ddots & \ddots & & & & & \\
 & & & & & 1 & -4 & 6 & -4 & 1 & & \\
 & & & & & & 1 & -4 & 6 & -4 & 1 & \\
 & & & & & & & 1 & -4 & 6 & -4 & 1 \\
 & & 0 & & & & & & 1 & -4 & 6 & -4 \\
 & & & & & & & & & 1 & -4 & 5
\end{bmatrix}$$

- Write your own CG and FOM (hand in code, don't use steal matlab's version).
- Solve $A\mathbf{x} = \mathbf{b}$ for $\mathbf{b} = \mathbf{e}_1$ for a few different sizes of A.
- Plot the residual history (versus iteration number) and comment/explain what you see.

Matrix Market Here we will utilize matrices from the online matrix market: `http://math.nist.gov/MatrixMarket/`. In particular, we will study the performance of various solvers for 2-D fluid flow in a driven cavity. Not to worry, these matrices are already built (by collection→SPARSKIT Collection→DRIVCAV)

- Write your own CG and GMRES code (with restart).
- Test CG versus GMRES for a number of element sizes (e.g. 5,20,30,40 or whatever runs in reasonable time) and for $Re = 0$ (to ensure symmetry)
- Test GMRES(j) with different restarts, j. First fix $Re = 0$ and varying first the element size. Second fix element size $N_e = 5$ and vary Re.
- Note: in these tests remember the helpful measures: clock time, convergence factor, residual/error history, and spectrum. Use these in making a convincing analysis of the different methods.

Performance Let $A = [103 \times 103]$. Contruct A so that it has 50 pairs of complex conjugate eigenvalues that are randomly distributed in the rectangle $[1, 2] \times [-\imath, \imath]$, plus 3 more eigenvalues of 4.0, 0.5, and -1.0. You can contstruct A by taking a random matrix V and letting $A = VDV^{-1}$. Let D have 50 blocks $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, so the the complex conjugate pairs are $\lambda = a \pm \imath b$. Also have a 3x3 diagonal block of the three remaining eigenvalues.

- Write your own BiCGSTAB code.
- Run GMRES, GMRES(10), BiCGSTAB, CGNE and plot all 4 on the same figure for:
  - residual-norm versus iterations
  - residual-norm versus mat-vecs (i.e. scale the iterations accordingling)
  - residual-norm versus time
  - error-norm versus iterations
  - error-norm versus mat-vecs (i.e. scale the iterations accordingling)
  - error-norm versus time
- Comment on the results.