

AM 194: Iterative Methods

Brown University
Homework, Set 1

Fall 2004
Due Wed. February 23, 2005

Programming Write three routines to be used in the following problems: Classical Gram-Schmidt, Modified Gram-Schmidt, and Householder (`cgs`, `mgs`, `house`, respectively). (**Note:** I suggest Matlab, but used whatever. Matlab also has `qr()` that can be tested against).

Initial test Mesh the interval $[-1, 1]$ with n grid points (e.g. $n = 128$). Construct the so-called *Vandermonde Matrix*: V . The columns of V are simply monomials $1, x, \dots, x^k$ (e.g. $k = 10$). So $V = [n \times k]$ and $V_{i,j} = x_i^j$ for $i = 1, \dots, n$ and $j = 1, \dots, m$. Now use one of the QR factorization above to factor A . Scale each column of Q by the last row of Q . Plot the columns of A and the columns of Q and compare. What have we done? (**Hint:** Go to wikipedia.org and look for “Legendre Polynomials”).

Algorithmic Stability We want to now test `cgs`, `mgs`, and `house` to see how they handle accurate computations. So let’s build a matrix A to factor, which has increasingly small singular values (a.k.a. eigenvalues for symmetric matrices). Steps:

- 1.1 get m (e.g. $m = 100$) singular values that decrease: $\sigma = 2^{-j}$, $j = 1, \dots, m$.
- 1.2 get two different orthogonal matrices (use `randn()` for a matrix and use a QR routine above to factor)
- 1.3 build $A = U * S * V$, where U and V are the orthogonal matrices and $S = \text{diag}(\sigma_j)$.
- 1.4 Find the QR factorization of A with `cgs`, `mgs`, and `house`

From this, plot R_j versus j for each case as well as σ_j for each case. Also look at $\|A - Q \cdot R\|_2$ and $\|Q^T Q - I\|_2$. Any difference in the cases? Comment on what you see (**Hint:** look up the matlab function `eps`).

Orthogonality Now, consider $A = \begin{bmatrix} 0.7 & 0.70711 \\ 0.70001 & 0.70711 \end{bmatrix}$. Find the QR factorization for `cgs`, `mgs`, `house`, and `qr`. Look at $\|A - Q \cdot R\|_2$ and $\|Q^T Q - I\|_2$. Comment.

Cost Finally, do a operation count analysis on the three algorithms.