

NAME: _____

AM 034

Brown University
FinalExam

Fall 2004
Wednesday December 15, 2004

No computers, calculators, books, notes, or crib sheet allowed. Write your **name** on each sheet of paper and start each new problem on a new page. For full credit, **show** all work.

- (20 pts.) 1. Consider a metal rod of length 2 and thermal diffusivity 1. The ends of the rod are held at a constant temperature of 1° and the initial temperature distribution in the rod is given by

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1; \\ 2 - x, & 1 \leq x \leq 2. \end{cases}$$

- (i) Find $u(x, t)$, explicitly.
(ii) Sketch $u(x, 0)$ and sketch (based on your intuition) $u(x, t)$ at a few selected times.

- (20 pts.) 2. Consider the 2π periodic function

$$f(x) = 2x - 2, \quad x \in [-\pi, \pi].$$

- (i) Sketch the function and determine if it is odd, even, or neither.
(ii) Find the Fourier series representation.
(iii) True or False: The partial sums $f_N(x)$ will converge in the 'maximum norm' (i.e. the pointwise maximum of the absolute value). Explain in one sentence why.

- (15 pts.) 3. Determine if $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is deficient (*hint*: show $p(\lambda) = -(\lambda - 1)^2(\lambda - 5)$).

- (15 pts.) 4. Find the matrix exponential e^{At} for $A = \begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}$.

- (15 pts.) 5. Discuss the stability properties of $x' = y$, $y' = -6x - y - 3x^2$. at each of its equilibrium points.

- (15 pts.) 6. Consider the system $x' = y$, $y' = -x - y$.

- (i) Can we use the Liapunov function $V(x, y) = x^2 + y^2$ to determine stability of the origin? (show)
(i) Can we use the Liapunov function $V(x, y) = 3x^2 + 2xy + 2y^2$ to determine stability of the origin? (show)