NAME: _____

AM 034

Brown University **Fall 2004**
Exam #1 **Friday October 8, 2004**

No computers, calculators, books, notes, or crib sheet allowed. Write your **name** on each sheet of paper and start each new problem on a new page. For full credit, **show** all work.

(15 pts.) **1.** Does $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$ have (choose one)

(i) no solutions?

(ii) a unique solution?

(iii) infinitely many solutions?

(15 pts.) **2.** Write

$$2y''(t) + 2y'(t) + y(t) = 0,$$

as a first order system: $\mathbf{x}' = A\mathbf{x}$. [**Bonus** (3pts): Classify the system (nodal source/sink, spiral source/sink, saddle, center, star, deficient node)]

(10 pts.) **3.** Define "degenerate matrix" (2-3 sentences).

(30 pts.) **4.** For

$$\begin{aligned} x_1' &= 3x_1 - 2x_2 \\ x_2' &= 2x_1 - 2x_2, \end{aligned}$$

with $x_1(0) = 1$ and $x_2(0) = 1$,

(i) Find the general solution

(ii) Find the unique solution that satisfies the IVP using e^{At}.

(iii) Classify the system (nodal source/sink, spiral source/sink, saddle, center, star, deficient node)

(iv) Sketch a simple phase portrait indicating relavant features.

(30 pts.) **5.** For $A = \begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & -1 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, if possible,

(i) find A^{-1}

(ii) find $det(A)$

(iii) find $\mathbf{x}^T A$