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AM 034

Brown University — **Fall 2004**

Homework, Set 7 — **Due Friday November 15, 2004**

7.1 Find the equilibrium points and the cycles of the following systems written in polar coordinates. Determine whether the equilibrium point at the origin, r=0, is asymptotically stable, neutrally stable, or unstable. Determine whether each cycle is a limit cycle, and if it is, whether it is attracting or repelling. Sketch the cycles and other orbits in the xy-plane by hand. Use arrowheads to show the direction of increasing time.

(a) $r' = 4r(4-r)(5-r)$, $\theta' = 1$

(b) $r' = r(1-r^2)(4-r^2)$, $\theta' = 1-r^2$

(c) $r' = r\cos(\pi r)$, $\theta' = 1$

7.2 Find all limit cycles and identify each as attracting or repelling.

(a) $x' = y - x(x^2+y^2)$, $y' = -x - y(x^2+y^2)$

(b) $x' = 2x - y - x(3-x^2-y^2)$, $y' = x + 2y - y(3-x^2-y^2)$

7.3 Verify that each system satisfies the conditions of the Van der Pol Cycle Theorem presented in class. For each value of μ, plot the limit cycle and some orbits that are attracted to it. Estimate the period of the x- and y-amplitude of each cycle; verify that the larger the value of μ, the longer the period and the larger the y-amplitude of the cycle but that the x-amplitude changes only a little.

(a) $x' = y - \mu(x^3 - 10x)$, $y' = -x$, $\mu = 0.1, 2$.

(a) $x' = y - \mu x(x^4 + x^2 - 1)/10$, $y' = -x$, $\mu = 0.1, 1$.

7.4 For the equations $x' = -y^3$, $y' = x^3$, give a reason why the linearized system does not determine the nature of the critical point $(0,0)$. Using a Liapunov function (try $V(x,y) = x^4 + y^4$), determine the stability.

7.5 Consider

$$x' = -y + xy^2 + 2x^3$$
$$y' = x + x^2y + 3y^3.$$

Characterize the critical point $(0,0)$ using the linearized system and also using a Liapunov function.