NAME: ________________________________

AM 034

Brown University | **Fall 2004**

Homework, Set 2 | **Due October 1, 2004**

2.1 For (a)-(f):

(i) Find the solution

(ii) Plot example trajectories

(iii) Plot the eigenlines

(iv) Classify as having a **nodal source or sink, saddle point, deficient/improper node, star/proper node, spiral source or sink, or a center point**

(v) Comment on the general behavior (1-2 sentences)

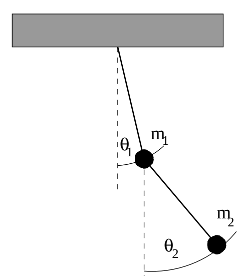
$$(a)\begin{cases} x' &= 3x-2y \\ y' &= 5x-3y \end{cases} \qquad (b)\begin{cases} x' &= x-4y \\ y' &= 5x-3y \end{cases} \qquad (c)\begin{cases} x' &= x+y \\ y' &= 4x-2y \end{cases}$$

$$(d)\begin{cases} x' &= 7x+6y \\ y' &= 2x+6y \end{cases} \qquad (e)\begin{cases} x' &= x-2y \\ y' &= y \end{cases} \qquad (f)\begin{cases} x' &= x \\ y' &= y \end{cases}$$

2.2 Consider the system $\begin{cases} x' &= y \\ y' &= -2x/t^2+2y/t \end{cases}$. Show that $X(t) = \begin{bmatrix} t^2 & t \\ 2t & 1 \end{bmatrix}$ is a fundamental solution set.

2.3 Suppose that one pendulum is suspended from another (see figure). It can be shown that the following linear system models the small-amplitude oscillations about the equilibrium position $x_1 = x_2 = x_3 = x_4 = 0$:

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -x_1 + \alpha x_3 \\ x_3' &= x_4 \\ x_4' &= x_1 - x_3, \end{aligned}$$



where $\alpha = (m_2/m_1)(1+m_2/m_1)^{-1}$ is the reduced mass. x_1 and x_2 represent the angles θ_1 and θ_2, while x_3 and x_4 represent the angular velocities, θ_1' and θ_2'.

(i) Build the general real-valued solution

(ii) For $m_1 = 1$, $m_2 = 1$, $\mathbf{x} = \begin{bmatrix} \sqrt{1/2} & 0 & 0 & 0 \end{bmatrix}$, find the unique solution and plot the solution of x_1 and x_2 versus t.

(iii) For $m_1 = 1$, $m_2 = 2$, $\mathbf{x} = \begin{bmatrix} \sqrt{1/3} & 0 & 0 & 0 \end{bmatrix}$, find the unique solution and plot the solution of x_1 and x_2 versus t.