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AM 033 — Applied Mathematics - I

Brown University **Fall 2003**

Homework, Set 9 **Due November 20**

9.1 Find the Laplace transforms of the following functions

$$\textbf{(a)}\ f(t) = \sin 3t \cos 3t, \qquad \textbf{(b)}\ f(t) = t[H(t-1) - H(t-4)],$$
$$\textbf{(c)}\ f(t) = 2tH(t-1), \qquad \textbf{(d)}\ f(t) = H(t-2) - H(3-t),$$

where $H(t)$ is the Heaviside function:

$$H(t) = \begin{cases} 1, & t > 0 \\ 1/2, & t = 0 \\ 0, & t < 0 \end{cases}$$

9.2 Use method of partial fractions to decompose the given rational functions into a sum of simpler expressions.

$$\textbf{(a)}\ \frac{2\lambda + 3}{(\lambda - 1)(\lambda - 2)^2}, \qquad \textbf{(b)}\ \frac{\lambda^2 + 5\lambda - 3}{(\lambda^2 + 16)(\lambda - 2)}.$$

9.3 Find the Laplace transform of the periodic function

$$f(t) = \begin{cases} \sin t, & \text{for } 0 < t < \pi, \\ 0, & \text{for } \pi < t < 2\pi, \\ f(t) = f(t + 2\pi), & \text{for all positive } t. \end{cases}$$

9.4 Find the convolution of two functions $f(t) = t$ and $g(t) = \sin 2t$.

9.5 Determine the Laplace transform, $Y(\lambda) = \mathcal{L}[y](\lambda)$, of the function $y(t)$ that is a solution of the given initial value problem.

$$\textbf{(a)}\ y'' + 4y' + 4y = t, \qquad y(0) = 1, \quad y'(0) = 0;$$
$$\textbf{(b)}\ y'' - 2y' = H(t-1), \qquad y(0) = 0, \quad y'(0) = 0.$$