NAME: ______________________________

AM 033 — Applied Mathematics - I

Brown University **Fall 2003**

Homework, Set 8 **Due Friday, November 7**

8.1 Find the solution to

$$y'' + y = H(t)H(\pi - t), \qquad \text{where } H(t) = \begin{cases} 1 & \text{if } t > 0, \\ 0 & \text{if } t < 0, \end{cases}$$

subject to the initial conditions $y(0) = y'(0) = 0$ and that y and y' are continuous at $t = \pi$.

8.2 Find the general solution of the differential equation written in factored form

$$(D-1)(D+1)y = 8\,e^{2t}, \qquad D = \frac{d}{dt},$$

applying sequential integration to the first order differential operators $D+1$ and $D-1$. That is, solve first the equation $(D-1)u = 8\,e^{2t}$ and then $(D+1)y = u$.

8.3 Use the method of undetermined coefficients to solve the following differential equations

(**a**) $(D^2+4)y = 8\,\sin^2 x$, (**b**) $y'' + 2y' - 3y = e^{-x}$,

(**c**) $y'' + 4y = t^2\,\sin 2t + (6t+7)\,\cos 2t$, (**d**) $(D^2 + 3D + 2)y = 1 + 3x + x^2$.

8.4 Verify that the given functions y_1 and y_2 satisfy the corresponding homogeneous equations; then find the general solution.

(**a**) $x^2\,y'' + xy' + (x^2 - 0.25)y = 3x^{3/2}\,\sin x$; $y_1(x) = x^{-1/2}\,\sin x$, $y_2(x) = x^{-1/2}\,\cos x$;

(**b**) $(1-x)\,y'' + xy' - y = \sin x$, $y_1(x) = e^x$, $y_2(x) = x$.