NAME: ____

AM 033 — Applied Mathematics - I

Brown University **Fall 2003**

Homework, Set 5 **Due October 17**

5.1 Consider the Cauchy problem for the ODE

$$y' = \frac{y^2 + 2ty}{3 + t^2}, \qquad y(0) = 0.5. \tag{1}$$

(a) Compute the approximate solution using Euler's Method and Improved Euler's Method for $h = 0.1$, 0.05, 0.025, and 0.0125 up to $t = 1.0$. For this portion, submit a plot of (i) Euler's versus Improved Euler's for $h = 0.1$ and (ii) Euler's method for $h = 0.1$, 0.05, 0.025, and 0.0125. It may be helpful to include the exact solution on these plots in a different color.

(b) Compute the error (global) at $t = 0.5$ and $t = 1.0$ and determine the "rate" at which the error is decreasing. Submit the value of the approximate solution calculated at $t = 0.5$ and $t = 1.0$ for each of the h values listed above as well as the rate calculated between each step size. [**Hint:** Let $E_h(t_n)$ be the global error (i.e. $y_n - \phi(t_n)$, where $\phi(t)$ is the exact solution to the given IVP (1)) at time t_n with step size h. If we assume $E_h \approx C\,h^p$, C is a positive constant, then $r = \frac{E_h}{E_{h/2}} \approx 2^p$. The ratios, r, can be determined and the rate p will be given by $p = \frac{ln(r)}{ln(2)}$.][**Advice:** This problem will be addressed on Wednesday, October 15th Room 265 CIT 6-7pm during the MATLAB tutorial.]

5.2 Consider initial value problem for the ODE

$$y' = 2y - 3t, \qquad y(0) = 1.0. \tag{2}$$

Find the approximate solution using the predictor-corrector method. Implement fourth order Adams-Bashforth (AB4) and fourth order Adams-Moulton (AM4) as a predictor and corrector (equations (6) and (10) in the text). Also, implement Euler's Method as a predictor and fourth order Adams-Moulton as a corrector. Use just one correction step.

(a) List the error at $t = 1.0$ for each implementation of the predictors listed above. Use $h = 0.05$, 0.025, and 0.0125.

(b) Plot the global error as a function of time for both of the predictors above.

(c) Compare the error using AB4 with 1 correction steps of AM4 versus 0 correction steps. A plot will suffice (for t=0..1).

5.3 Show that Euler's method fails using certain time steps and succeeds using other time steps for the following IVP:

$$y' = 10(1 - y)y; \qquad y(0) = 1.25. \tag{3}$$

[**Hint:** try $h > 0.2$ and $h < 0.1$]