NAME: _____

AM 033 — Applied Mathematics - I

Brown University **Fall 2003**
Homework, Set 4 **Due October 2**

4.1 Initially, 2 kg of salt are dissolved in a 200-liter container and a salt solution with concentration 100 gram/liter runs into the tank at a rate of 5 liter/min. The solution inside the tank is kept well-stirred and drops off at the same rate as that at which it flows in. Write down the differential equation for the amount of salt and determine this amount in the tank at any time.

4.2 Let us consider a container or tank with liquid. Suppose there is a hole in a tank with the area A that allows liquid to escape. We denote $S(y)$, the horizontal cross-section area of the container at depth y. The rate dy/dt at which the volume of liquid diminishes is given by the **Torricelli's Equation**

$$S(y)\,\frac{dy}{dt} = -k\,A\,\sqrt{2gy},$$

where g is the acceleration due to gravity and k is a constant that characterizes the viscosity of the liquid. For example, water has $k = 0.6$ and oil has a k of approximately 0.4.

Initially, a cylindrical tank 2 m high with a radius of 1 m is full of oil. The tank begins to drain when a circular plug of radius 0.2 m is removed from the bottom of the tank. Determine the time when the tank will be half-full of oil.

4.3 Find a continuous solution of the following Cauchy problem $y' + 2y = f(x)$, $y(0) = 0$, where

$$f(x) = \begin{cases} 1, & 0 \le x \le 3; \\ 0, & x > 3. \end{cases}$$

4.4 Solve the following Bernoulli equation $t^2\,y' + 2ty - y^3 = 0$, $t > 0$.

4.5 Solve the equations with the dependent variable missing.

$$\textbf{(a)}\ \ 4y'' - (y')^2 + 4 = 0; \qquad \textbf{(b)}\ \ y'' + x(y')^3 = 0.$$

4.6 Solve the equations with the independent variable missing.

$$\textbf{(a)}\ \ y'' + 2y(y')^3 = 0; \qquad \textbf{(b)}\ \ y^2\,y'' + y' + 2y(y')^2 = 0.$$