

NAME: _____

AM 033 — Applied Mathematics - I

Brown University
Homework, Set 3

Fall 2003
Due September 26

3.1 Any body moving through a fluid such as water or air creates a drag force that tends to retard its motion. Calculations predict and experiments confirm that in air, the drag force on a parachutist/skydiver or a mass fired into earth can be well approximated by a force, called *Newtonian damping*, that is proportional to the square of the magnitude of its velocity v^2 , namely, $F = -k v^2$, with the Newtonian damping constant k . Balancing the forces of acceleration, gravity, and air resistance leads to the nonlinear differential equation of motion

$$m \frac{dv}{dt} = -mg + k v^2, \quad \text{or} \quad \frac{dv}{dt} = -g + \frac{k}{m} v^2$$

with the homogeneous initial condition $v(0) = 0$. **Note:** Maple may be quite useful in assisting you with this problem.

If the mass of the equipped parachutist is 100 kg, the the Newtonian damping constant k is about 0.2 kg/m. Typical questions to be addressed in the analysis of the “parachute problem” include the following:

- (a) Training jumps for the Parachute team begin 1,200 meters above ground level (AGL). What is the terminal velocity ($\lim_{t \rightarrow \infty} v(t)$)?
- (b) The free-fall portion of the jump lasts about 10 seconds; free-falls longer than 13 seconds are grounds for removal from the team. What is the velocity of the skydiver after 10 seconds of free-fall? What is his/her altitude AGL?
- (c) If a malfunction occurs with the main chute, almost 6 seconds will be required to recognize and react to the problem and to deploy the reserve parachute. What is the velocity and the altitude of the skydiver after 16 seconds of free-fall?
- (d) The parachute requires approximately 3.2 seconds to fully deploy from the time the ripcord is pulled. For simplicity, we ignore this time and assume that the chute is deployed immediately. The Newtonian damping constant is not actually a constant but is of the following form:

$$k(t) = \begin{cases} k_1, & \text{if } t < t_d, \\ 7k_1, & \text{if } t > t_d, \end{cases}$$

where $k_1 = 0.2$. What is the velocity at impact if the parachute was deployed after 10 seconds?

- (e) The landing velocity should be no worse than a free-fall from 1.5 meters wall-between 4.6 and 5.2 m/sec. What is the latest time that the parachute can be open while keeping the impact velocity below a specified threshold?

3.2 Solve the initial value problems

- (a) $(9x^2 + y - 1)dx = (4y - x)dy$, $y(1) = 0$;
- (b) $y' = e^x + 2y$, $y(0) = -1$;
- (c) $3x^{-1}y^2 dx - 2y dy = 0$, $y(4) = 8$;
- (d) $(2x e^x - y^2) dx + 2y dy = 0$, $y(0) = \sqrt{2}$.