NAME: _____

AM 033 — Applied Mathematics - I

Brown University **Fall 2003**
Homework, Set 2 **Due September 19**

2.1 Differentiate both sides of the given equation $\sin^2 y - \tan x = C$ to eliminate the arbitrary constant (denoted by C) and to obtain the associated differential equation.

2.2 Let $y(x)$ be a solution for the ODE $y''(x) = x\,y(x)$ that satisfies the initial conditions $y(0) = 1$, $y'(0) = 0$. (You will learn in this course that exactly one such solution exists.) Calculate $y''(0)$. The ODE is, by its nature, an equation that is meant to hold for *all* values of x. Therefore you can take the derivative of the equation. With this in mind, calculate $y'''(0)$ and $y^{(4)}(0)$.

2.3 A particle moves along the abscissa so that its instantaneous acceleration is given as a function of time t by $a = 2 - 3t^2$. At times $t = 1$ and $t = 4$, the particle is located at $x = 5$, and $x = -10$, respectively. Set up a differential equation and associated conditions describing the motion.

2.4 Geological dating of rocks is done using potassium-40 rather than carbon-14 because potassium has a longer half-life 1.28×10^9 (the half-life is the time required for the quantity to be reduced by one half). The potassium decays to argon, which remains trapped in the rocks and can be measured. Derive the differential equation that the amount of potassium obeys.

2.5 6 out of the 1000 passengers, crew, and staff that board a cruise ship, has the flu. After one day of sailing the number of infected people has risen to 12. Assuming that the rate at which the flu virus spreads is proportional to the product of the number of infected individuals times the number not yet infected individuals. Determine how many people will have the flu at the end of the 7 day cruise.

2.6 Find all functions that have the property of being squared with itself plus the square of its derivative results to 1.

2.7 Formulate the given differential equation $2xy\,y' = (3x^2 - 2y^2)$ as a differential equation with a homogeneous right-hand side function. Then solve this equation subject to the initial condition $y(0) = 1$.

2.8 Solve the following equation with linear coefficients: $(x + y + 1)\,dx + (y - x - 3)\,dy = 0$.