NAME: _____

AM 033 — Applied Mathematics - I

Brown University **Fall 2003**
Homework, Set 11 **Due Thursday, December 11**

11.1 Find solution of the given initial value problem.

(**a**) $x^2y'' - xy' + y = 0, \qquad y(1) = 1, \qquad y'(1) = 2;$
(**b**) $(x+1)^2\,y'' + 8(x+1)y' + 12y = 0, \qquad y(1) = 1, \qquad y'(1) = -2;$

11.2 Identify the singular points of the given differential equation.

(**a**) $(1-t^2)\,y'' + ty' + (\tan t)\,y = 0,$ (**b**) $(1+\ln|t|)\,y'' + t^2\,y' + y = 0.$

11.3 For the Chebyshev differential equation

$$(1-x^2)\,y'' - xy' + \alpha^2\,y = 0,$$

where α is a constant, determine two linearly independent solutions in powers of x for $|x| < 1$. Find a polynomial solution for $\alpha = 0$, 2, 3, and 4.

11.4 Use the series method to determine the recurrence relation for the coefficients, a_n, of the solution $y(x) = \sum_{n\geq 0} a_n x^n$ to the following first order differential equations:

(**a**) $y' - y = x^2,$ (**b**) $(1-x)\,y' - y = 2x.$

11.5 For the given differential equations

(**a**) $x^2(1-x)\,y'' - (x-2)\,y' + (x-3)\,y = 0,$ (**b**) $x^2(1-x^2)\,y'' - (2/x)\,y' + (x-4)\,y = 0,$
(**c**) $(1-x^2)\,y'' - x(1-x)\,y' + (x^2-4)\,y = 0,$ (**d**) $y'' + (\ln|x|)\,y' - xy = 0,$

find all singular points and determine whether each one is regular or irregular.