

DUAL DOMAIN INTERACTIVE IMAGE RESTORATION : BASIC ALGORITHM

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ABSTRACT

This paper describes a new fast, iterative algorithm for interactive image noise removal. Given the locations of noisy pixels and a prototype image, the noisy pixels are to be restored in a natural way. Most existing image noise removal algorithms use either frequency domain information (e.g low pass filtering) or spatial domain information (e.g median filtering or stochastic texture generation). However, for good noise removal, both spatial and frequency information must be used. The existing algorithms that do combine the two domains (e.g Gerchberg-Papoulis and related algorithms) place the limitation that the image be band limited and the band limits be known. Also, some of these may not work well when the noisy pixels are contiguous and numerous.

Our algorithm combines the spatial and frequency domain information by using projection onto convex sets (POCS). But unlike previous methods it does not need to know image band limits and does not require the image to be band limited. Results given here show noise removal from images with texture and prominent lines. The detailed textures as well as the pixels representing prominent lines are created by our algorithm for the noise pixels. The algorithm is fast, the cost being a few iterations (usually under 10), each requiring an FFT, IFFT and copying of a small neighborhood of the noise.

1. INTRODUCTION

Existing noise removal algorithms have difficulty with noise which (i) consists of many contiguous pixels and (ii) is in a textured area of image or areas with prominent lines. Note that by *texture* we mean not only small stochastic texture but also small patterns like fabric texture as shown in middle row of Fig. 3. In addition there can be prominent systematic lines or lines placed randomly in the image. The brick wall in the top row of Fig. 3 and the stone wall in the bottom row of Fig. 3 are examples. Our algorithm for removal of such noise is based on the theory of projections onto convex sets. It is a fast iterative algorithm alternating between the frequency and spatial domains, using the available information in each domain. Unlike previous POCS based or other iterative algorithms for noise removal, we place no restrictions on the band limits of the image. For our algorithm, the image does not have to be band limited and if it is, the band limits don't have to be known.

The pixels determined by the algorithm to replace the noise are (a) as sharp as the surrounding area (b) maintain continuity of prominent lines running across the noise pixels and (c) have a texture matching the surrounding texture. While some previous algorithms were able to remove such noise from images with stochastic texture or small regularly patterned textures, ours works

on those as well as the more difficult cases of systematic or randomly placed prominent lines. To our knowledge it is the first application of POCS for interactive image noise removal. POCS allows the combination and use of frequency and spatial domain information in an extendible way. Our algorithm works even when the noisy pixels are contiguous and numerous. The image intensity can also be non-uniform. But for very large changes in intensity across an image, an extension to our algorithm is described in [1]. [1] also gives a more detailed description and usage of the algorithm described here.

2. PREVIOUS WORK

Low-pass filtering is a simple method for hiding small isolated noise in smooth or blurred areas of images. It is not possible however, to retain sharpness of lines and other details. Median and similar non-linear filters do better but not when many contiguous pixels are missing. Another simple technique – copying pixels from another area of the image may work for stochastic type texture with no prominent lines. For images with prominent lines manual alignment of lines is time consuming and error prone.

Recent statistical texture synthesis algorithms [2] that use second order statistics have shown remarkable results but don't work well in images with long range structure due to computational cost that increases prohibitively with the distance between correlated pixels. An example of an image with long range structure or large features is the brick wall image in the top row of Fig. 3. Another problem for such methods is texture with prominent lines and irregular features like the stone wall image in the third row of Fig. 3.

It is possible to use multi-resolution directional filters and then work with only single order statistics (histograms) as in [3]. This could be considered a spatial and frequency domain algorithm. However this method works only for stochastic texture or small regular texture. In addition [3] is not a noise removal algorithm. It is designed for generating large texture areas from sample images. It cannot be used to generate pixels that maintain continuity of prominent lines crossing noise pixels while retaining the noise free pixels.

Recent work on image restoration based on matrix algorithms [4] has been used for noise removal using smaller matrices than previously possible. However, [4] does not show examples of many contiguous noise pixels and indicates that the method may not work well in such cases. In addition, the image must be band limited and it's band limits must be known. Previous algorithms related to Gerchberg-Papoulis algorithm [5, 6] also place this restriction.

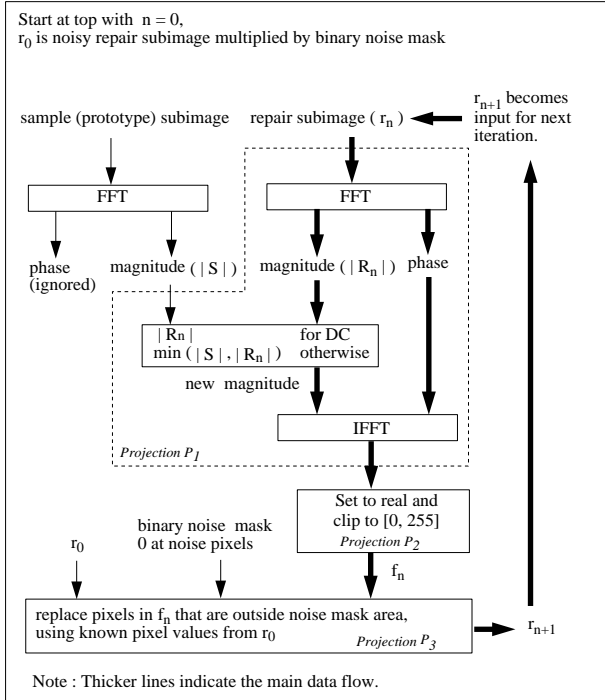


Figure 1: Our algorithm for image noise removal.

In summary, (i) frequency domain algorithms such as low pass filtering can capture global structure of the image but lose local control (line continuity, sharpness), (ii) spatial domain algorithms like median filtering and statistical texture generation [2] have no information about global structure and work purely with local spatial information due to computational constraints, and (iii) previously known methods for combining the two domains like Gerchberg-Papoulis [6, 4] or related algorithms or Ferreria’s algorithm [5] require that the image be band limited and the band limits be known. In addition they may not work well when the noisy pixels are contiguous and numerous.

The well known reconstruction method of projections onto convex sets (POCS) allows the use of *any* information about the image that can be represented as a closed convex set. For details on this see the classic Youla and Webb paper [7]. However, no projections based algorithms that can remove image noise under the conditions described in the introduction have been reported so far.

3. OUR ALGORITHM

Fig. 1 gives a flowchart of our algorithm which is an iterative, prototype based algorithm. The algorithm uses sample (prototype) and repair subimages which have been selected interactively. See Fig. 2 for examples of sample and repair subimages which are shown as dark gray patches. The noisy pixels are indicated by the user interactively by creating a mask that covers the noise. These noisy masks covering the noisy pixels are shown as the black lines in Fig. 2. The algorithm replaces each noise mask 0 pixel with a reconstructed value. Thus the mask can be considered to be the noise in what follows.

The sample subimage (used as a prototype) contributes information about the global structure of the image area around the

noise. This is done by using the frequency domain information of the sample subimage. The repair subimage is the one which has the noise. The neighborhood of the noise in the repair subimage contributes the local spatial information. These two pieces of information are put together by the algorithm.

Note that simply *copying* the sample subimage into the repair subimage would not work. Consider for example the sample subimage labeled s_a in Fig. 2. Clearly, copying this subimage directly into the areas marked as r_a would cause a break in the line of “cement” running horizontally across the two subimages. This is a key point, that the two subimages are similar but the prominent lines in these are *not* aligned. The same can be seen in the subimages in the bottom image in Fig. 2.

Here we will briefly summarize the algorithm given in the flowchart. The projection \mathcal{P}_1 starts with a repair and sample subimage and applies a Fourier transform followed by an operation in frequency domain ending finally with an inverse Fourier transform. The result of \mathcal{P}_1 may not be real and may have real part which is outside $[0, 255]$. It is made real and each pixel restricted to $[0, 255]$ by the projection \mathcal{P}_2 . Finally \mathcal{P}_3 applies the spatial domain constraint, which is to copy the noise free pixels without change, completing one iteration of the algorithm. The result of \mathcal{P}_3 will be used as the repair subimage input for the next cycle. In the current implementation the user sets the number of iterations. It is easy to implement other termination criteria.

The algorithm is fast because it usually converges in under 10 iterations and each iteration requires 1 FFT, 1 IFFT and copying, all performed on a small neighborhood of the noise and *not* on the entire image.

3.1. Phase reconstruction

When the pixels of an image are translated (with wrap around), the spectrum magnitude stays unchanged (note that when we say *translate* we do not mean moving subimage locations within the image, but rather moving the pixels of a subimage). The repair subimage can be thought of as a translated, noisy version of the sample subimage. See for example the sample subimage s_a and the repair subimage r_a in Fig. 2. The thick horizontal line in s_a is nearly in its middle, but not so in r_a . If the sample subimage was an exact translated version of the repair subimage without the noise then removing the noise could be thought of as shifting the sample subimage to match the features in the noisy repair subimage.

Translating the sample subimage amounts to adjusting its phase, hence the connection with phase reconstruction algorithms of astronomy [8] and other fields. The difference is that in our formulation of the problem, only approximate information about the desired spectrum magnitude and some information (pixels outside the noise area) about the spatial domain nature of the image are available. From these, the phase must be reconstructed and the magnitude must be improved.

3.2. Constraints and projections used

The key step in using POCS is finding the right convex sets and defining the projections correctly. The constraints or closed convex sets (\mathcal{C}_i) and projections (\mathcal{P}_i) onto these are given below. After defining these we give a brief intuitive justification for selection of these convex sets and projections. In these equations r_0 is the given noisy repair subimage multiplied by the binary noise mask and s is the sample subimage. Let $R = \text{FFT}(r)$ and $S = \text{FFT}(s)$.

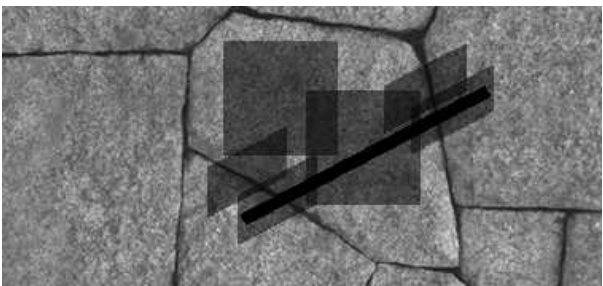
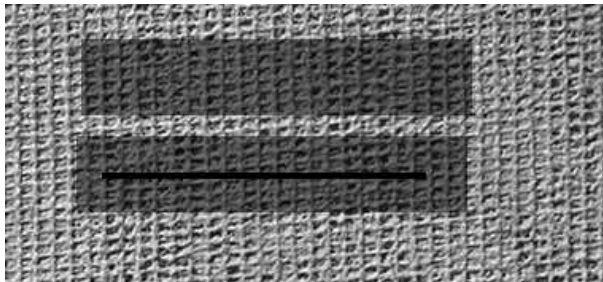
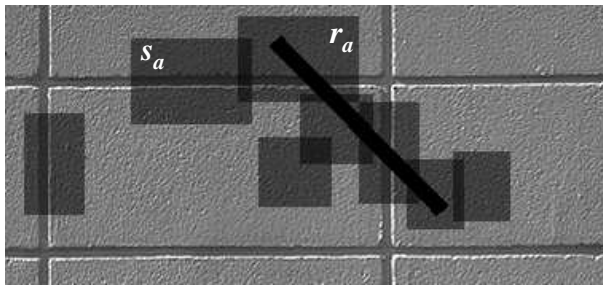


Figure 2: Repair and sample subimages for Fig. 3. Black line is noise mask, sample and repair subimages are shown as dark patches. Features in sample and repair don't have to be aligned e.g the thick horizontal line in s_a and r_a .

Then define

$$\mathcal{C}_1 = \{r : |R(u, v)| \leq |S(u, v)| \text{ for } (u, v) \neq (0, 0)\}.$$

To define $\mathcal{P}_1(r)$, let

$$M = \begin{cases} \min(|R(u, v)|, |S(u, v)|) & \text{if } (u, v) \neq (0, 0) \\ |R(0, 0)| & \text{otherwise} \end{cases}$$

Then define $\mathcal{P}_1(r) = \text{IFFT}(M e^{i \text{phase}(R)})$. The next two convex sets and corresponding projections are simpler and can be written as follows. Define

$$\mathcal{C}_2 = \{r : r(j, k) \text{ is real and } 0 \leq r(j, k) \leq 255\}.$$

Then define the projection operation $\mathcal{P}_2(r)$ as : Set imaginary component of $r(j, k)$ to 0, clip to $[0, 255]$. Define

$$\mathcal{C}_3 = \{r : r(j, k) = r_0(j, k) \text{ for } (j, k) \text{ in noise-free area}\}.$$

Now let w be a binary mask with 0 at noise locations. Then $\mathcal{P}_3(r) = r(1 - w) + r_0 w$. The algorithm can now be written as

$$\begin{aligned} r_0 &= \text{given noisy repair subimage} \\ r_{n+1} &= \mathcal{P}_3 \mathcal{P}_2 \mathcal{P}_1 r_n. \end{aligned}$$

An intuitive justification for the selection of these convex sets and projections is as follows. Noise of the type discussed in this paper introduces high magnitude values in the spectrum. In most real images, the sample spectrum can only be used as a hint. Using the sample spectrum magnitude while ignoring the repair spectrum magnitude would lose the known information of the repair subimage. Also, such a constraint would not be a convex set. Hence the minimum is used, which reduces the effect of noise while retaining some information of the repair subimage spectrum.

The reason for retaining the DC component is that taking the minimum for DC can cause the overall brightness of the image to be reduced. This also allows the algorithm to be somewhat robust to differences in the intensity of repair and sample subimages (for very severe differences in intensity, an extension is given in [1]). We rely on the human operator to select a sample that is similar to the repair subimage. Note that \mathcal{C}_1 is closed and convex because it is a special case of $\mathcal{C}_M(\Delta)$ on pp. 86 of [7] which is proven to be closed and convex there. Proof that the other \mathcal{C}_i are closed and convex is simple. The justification for using \mathcal{C}_2 is that we know that pixel values must be real and between 0 and 255. \mathcal{C}_3 is justified because the area outside noise pixels is known to be correct. The previous frequency domain operation \mathcal{P}_1 affects the repair subimage globally. Thus the pixels known to be noise free should be replaced using the original known values in each iteration.

4. RESULTS

Fig. 3 shows noise removal from images with stochastic and regular textures and prominent systematic and random lines. The black line running across the first image of each pair in Fig. 3 is the noise mask which is treated as the noise by the algorithm. The images on the right are the reconstructed images after 10 iterations of our algorithm. The dark patches in Fig. 2 show the location of sample and repair subimages.

The detailed textures as well as the pixels representing prominent lines are created as replacement for the noise pixels by our algorithm. Note that simple copying of say, s_a into r_a would have caused a break in the thick horizontal line in these subimages (“cement” between the bricks).

5. CONCLUSIONS AND FURTHER WORK

A fast iterative algorithm for combining frequency and spatial domain information for image noise removal has been described. With a few exceptions, most previous algorithms like low-pass filtering, median filtering and statistical texture generation [2], have worked in only one domain. The few previous algorithms for combining frequency and spatial domain information [6, 4] require the image to be band limited, require that the band limits be known and may not work well when many contiguous pixels are noisy. Our algorithm does not place this limitation.

The algorithm described is extendible in a clean way. For example, any constraints or information about images that can be formulated as a closed convex set can be added easily. Thus the effectiveness of a novel interactive use of POCSS has been demonstrated. The work presented here involves four important concepts – (i) sample subimages as prototypes for repair subimages (ii) projection onto convex sets for noise removal, (iii) combining frequency and spatial domain information and, (iv) the new constraint \mathcal{C}_1 and the projection \mathcal{P}_1 described in Section 3.2.

These allow the reconstruction to generate texture matching

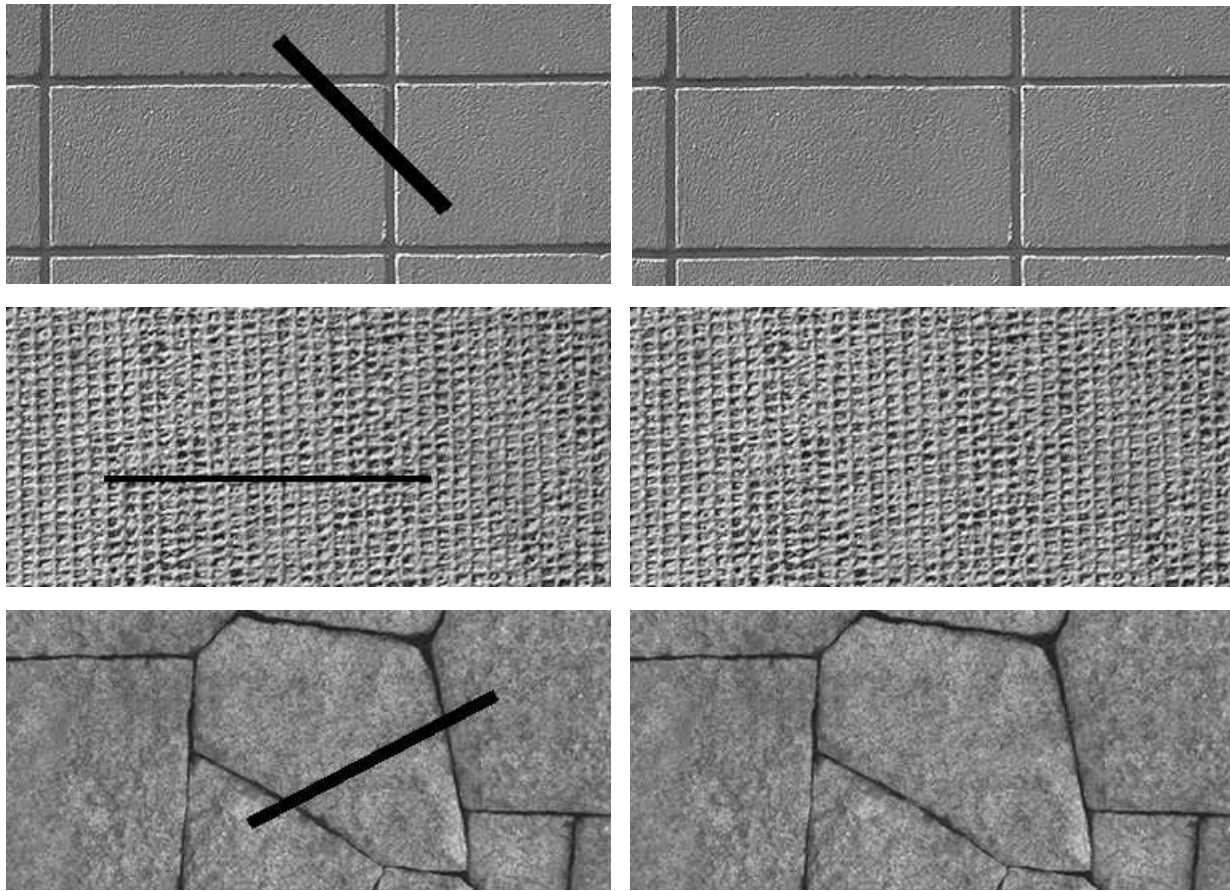


Figure 3: Removal of noise from images with large regular (bricks), small regular (fabric) and large irregular (stones) features. Left column is noisy image, right column shows result after 10 iterations of our algorithm. Images are 377×222 , and noise approx. 9, 4 and 8 pixels wide.

the surroundings, maintain sharpness and maintain line continuity across the replaced pixels even when the noise pixels to be replaced are contiguous and many in number. An important application of this algorithm is in the field of film post production for removing wires used in special effects scenes. Another important application is in restoring old films and photographs that have become scratched.

An extension that allows the repair and sample subimage to differ substantially in shading and another that makes the image noise mask soft edged are described in [1]. A simple color result applying the algorithm to R, G and B separately is also shown there. Other improvements in implementation are obvious. These include brush like strokes for painting over the noisy area, removal of perspective distortion between the sample and repair subimages. A multi-frame extension that uses computation done in one frame to help in repair of subsequent or previous frames is another area for future work. Our application to color images is naive and a better algorithm for color images is also needed. All these are issues that we are currently working on.

6. REFERENCES

[1] Hirani, A.N. and Totsuka, T., "Combining Spatial and

Frequency Domain Information for Fast Interactive Image Noise Removal", Proc. SIGGRAPH 96, 1996.

[2] Malzbender, T. and Spach, S., "A Context Sensitive Texture Nib", Communicating with Virtual Worlds, ed. Thalmann, N.M. and Thalmann, D., Springer Verlag, 1993.

[3] Heeger, D.J. and Bergen, J.R. "Pyramid-Based Texture Analysis/Synthesis", Proc. SIGGRAPH 95, 229-238, 1995.

[4] Ferreira, P.J.S.G. and Pinho, A.J., "Errorless Restoration Algorithms for Band-Limited Images", Proc. 1994 IEEE Intl. Conf. Im. Proc. (ICIP-94), vol. III, 1994.

[5] Ferreira, P.J.S.G., "Interpolation and the Discrete Papoulis-Gerchberg Algorithm", IEEE Trans. Sig. Proc., vol. 42, No. 10, October 1994.

[6] Papoulis, A., "A New Algorithm in Spectral Analysis and Band-Limited Extrapolation", IEEE Trans. Cir. & Sys., vol. 22, No. 9, 735-742, 1975.

[7] Youla, D.C. and Webb, H., "Image Restoration by the Method of Convex Projections: Part 1 - Theory", IEEE Trans. Med. Im., vol. MI-1, No. 2, 1982.

[8] Fienup, J.R., "Phase Retrieval Algorithms: a Comparison", Applied Optics, vol. 21, No. 15, 1982.