CS273: Theory of Computation, Summer 2008

Second Midterm — July 16, 2008, 9:00am-10:15am

Name:	**Net ID:**

Instructions

1. This is a closed-everything exam. No notes or electronics of any kind are allowed.
2. Print your full name and your NetID in the boxes above.
3. Print your name at the top of every page.
4. Please write clearly and legibly. If we can't read your answer, we can't give you credit.
5. You have 75 minutes to complete the exam; please plan accordingly. Do not spend too much time on any one of the problems. Problems do not necessarily appear in order of difficulty.
6. You may write "I Don't Know" on any problem to receive 20% of the total credit. If you would like to answer a problem with "I Don't Know", be sure to cross out any partial solution that you may have started writing.
7. There are 146 points available on the exam, but we will record a maximum score of 130 points.[1]

#	1	2	3	4	5	6	7	8	9	10	Total
Points	5	16	15	10	15	15	15	20	20	15	146
Score											/130

[1] So, there are $\sum_{i=130}^{146} \binom{146}{i} = 987,930,261,553,815,528,005$ ways to earn a perfect score.

1. **[5 points]** In the following definition of a PFA, the type of the transition function δ is missing. Fill in the blanks.

> A <u>PDA</u> is a 6-tuple $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where
> - Q is a finite set of states,
> - Σ is the input alphabet,
> - Γ is the stack alphabet,
> - δ : ____________________ $\longrightarrow$ ____________________ is the transition function,
> - $q_0 \in Q$ is the start state, and
> - $F \subseteq Q$ is a set of accept states.

2. **[16 points; 2 per part]** Categorize each of the following languages by checking the appropriate box. You do not need to show your work. The "I don't know" rule does not apply to this problem. (You should check exactly one box in each row.)

Language	Regular	Context-free and non-regular	Not context-free
$\{a^i b^j c^k d^l \mid i = j,\, k = l\}$			
$\{a^i b^j c^k d^l \mid i = k,\, j = l\}$			
$\{a^i b^j c^k d^l \mid i = l,\, j = k\}$			
$\{a^n a^n \mid n \geq 0\}$			
$\{a^n b^m c^k \mid n + m = k\}$			
$\{a^n b^m c^k \mid n + k = m\}$			
$\{x\#y \mid x$ is a substring of $y\}$			
$\{w a^n b^n w^R \mid w \in \{a, b\}^*, n \geq 0\}$			

3. **[15 points; 5 per part]** Are the following statements true or false? Give a brief argument or counterexample:

   (a) If A and B are context-free, then $A \cap B$ is context-free.

   (b) If A is context-free and B is regular, then $A - B$ is context-free. (Recall that $A - B = \{w \mid w \in A \text{ and } w \notin B\}$.)

   (c) If A is context-free and $B \subseteq A$, then B is regular.

4. **[10 points]** Prove that the context-free languages are closed under string reversal. (In other words, show that if A is context-free, then $A^R = \{w^R \mid w \in A\}$ is also context-free.) *[Hint: Modify a CFG for A to obtain a CFG for A^R.]*

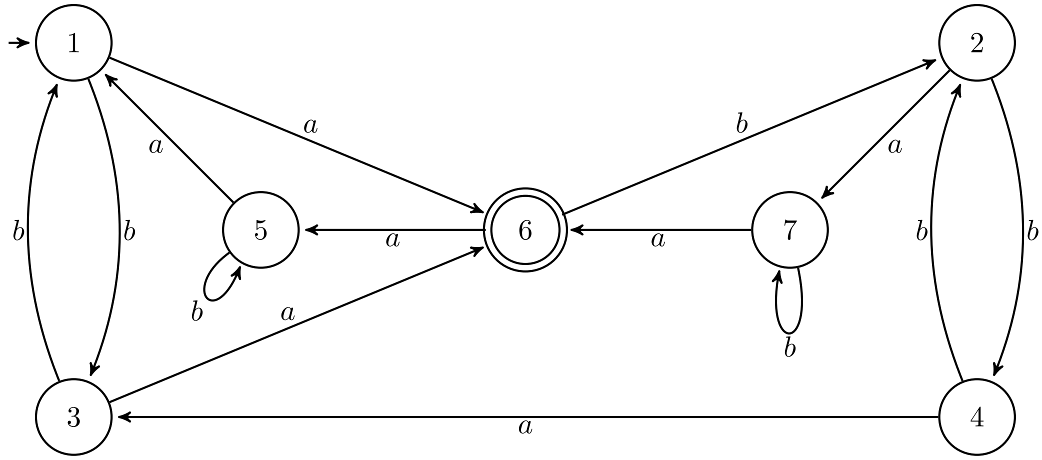
5. **[15 points]** Convert the following grammar to a GPDA. The start variable is S.

$$S \rightarrow baS \mid T$$
$$T \rightarrow Ta \mid ST \mid \varepsilon$$

6. [**15 points**] Convert the following grammar to Chomsky normal form. The start variable is S.

$$S \to STSa \mid \varepsilon$$
$$T \to bT \mid c$$

7. [**15 points**] Let M be the following DFA.



(a) Use the following table to find which pairs of states in M are distinguishable.

1	1						
2		2					
3			3				
4				4			
5					5		
6						6	
7							7

(b) List the sets of equivalent states.

(c) Give a state diagram for a minimal DFA that is equivalent to M.

8. **[20 points; 10 per part]** Give context-free grammars that generate the following languages. (You do not need to explain your grammars.)

   (a) $\{a^m b^n \mid m > n\} \cup \{b^m a^n \mid m > n\}$

   (b) $\{a^i b^j c^j d^k e^k f^i \mid i, j, k \geq 0\}$

9. **[20 points; 10 per part]** Give state diagrams of pushdown automata that recognize the following languages. (You do not need to explain your PDAs.)

   (a) $\{w \in \{a, b, c\}^* \mid \#a(w) = \#b(w) \text{ or } \#b(w) = \#c(w)\ \}$

   (b) $\{a^n b^m \mid n = 3m\}$

10. **[15 points]** Let $A = \{a^\ell b^m c^n \mid m > \ell \text{ and } m > n\}$. Use the pumping lemma to prove that A is not context-free.

For reference, here is the context-free *pumping lemma game* (for language A):

(a) Adversary picks a number $p \geq 0$.

(b) You pick a string $s \in A$, such that $|s| \geq p$.

(c) Adversary breaks s into $s = uvxyz$, such that $|vxy| \leq p$ and $vy \neq \varepsilon$.

(d) You pick a number $i \geq 0$. If $uv^ixy^iz \notin A$, then you win.

If you can describe a strategy in which you always win, then A is not context-free.

(scratch paper)