CS 373: Intro to Theory of Computation
Spring 2009 Mock Final Exam

INSTRUCTIONS (read carefully)

- Print your name and netID here and netID at the top of each other page.

NAME:

NETID:

- The exam contains 12 pages and 11 problems. Make sure you have a complete exam.
- You have three hours.
- The point value of each problem is indicated next to the problem and in the table below.
- It is wise to skim all problems and point values first, to best plan your time. If you get stuck on a problem, move on and come back to it later.
- Points may be deducted for solutions which are correct but excessively complicated, hard to understand, hard to read, or poorly explained.
- This is a closed book exam. No notes of any kind are allowed. Do all work in the space provided, using the backs of sheets if necessary. See the proctor if you need more paper.
- Please bring apparent bugs or unclear questions to the attention of the proctors.

Problem	Possible	Score
1	9	
2	18	
3	6	
4	5	
5	8	
6	6	
7	10	
8	8	
9	8	
10	6	
11	8	
12	8	
Total	100	

Problem 1: True/False (18 points)

Completely write out "True" if the statement is necessarily true. Otherwise, completely write "False". Other answers (e.g. "T") will receive credit only if your intent is unambiguous. For example, "$x + y > x$" has answer "False" assuming that y could be 0 or negative. But "If x and y are natural numbers, then $x + y \geq x$" has answer "True". You do not need to explain or prove your answers.

1. Let M be a DFA with n states such that $L(M)$ is infinite. Then $L(M)$ contains a string of length at most $2n - 1$.

2. Let $L_w = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ accepts } w\}$, where w is some fixed string. Then there is an enumerator for L_w.

3. The set of undecidable languages is countable.

4. For a TM M and a string w, let $CH_{M,w} = \{x \mid x \text{ is an accepting computation history for } M \text{ on } w\}$. Then $CH_{M,w}$ is decidable.

5. The language $\{\langle M, w \rangle \mid M \text{ is a linear bounded automaton and } M \text{ accepts } w\}$ is undecidable.

6. The language $\{\langle G \rangle \mid G \text{ is a context-free grammar and } G \text{ is ambiguous}\}$ is Turing-recognizable.

7. Context-free languages are closed under homomorphism.

8. The modified Post's correspondence problem is Turing recognizable.

9. There is a bijection between the set of Turing-recognizable languages and the set of decidable languages.

Problem 2: Classification (20 points)

For each language L described below, classify L as

- **R**: Any language satisfying the information must be regular.
- **C**: Any language satisfying the information must be context-free, but not all languages satisfying the information are regular.
- **DEC**: Any language satisfying the information must be decidable, but not all languages satisfying the information are context-free.
- **NONDEC**: Not all languages satisfying the information are decidable. (Some might be only Turing recognizable or perhaps even not Turing recognizable.)

For each language, circle the appropriate choice (**R**, **C**, **DEC**, or **NONDEC**). If you change your answer be sure to erase well or otherwise make your final choice clear. **Ambiguously marked answers will receive no credit.**

1. **R** **C** **DEC** **NONDEC**
   $L = \{\langle M \rangle \mid \text{M is a linear bounded automaton and } L(M) = \emptyset\}$.
2. **R** **C** **DEC** **NONDEC**
   $L = \{ww^Rw \mid w \in \{a,b\}^*\}$.
3. **R** **C** **DEC** **NONDEC**
   $L = \{w \mid \text{the string } w \text{ occurs on some web page indexed by Google on May 3, 2007}\}$
4. **R** **C** **DEC** **NONDEC**
   $L = \{w \mid w = x\#x_1\#x_2\#...\#x_n \text{ such that } n \geq 1 \text{ and there is some } i \text{ for which } x \neq x_i\}$.
5. **R** **C** **DEC** **NONDEC**
   $L = \{a^ib^j \mid i + j = 27 \bmod 273\}$
6. **R** **C** **DEC** **NONDEC**
   $L = L_1 \cap L_2$ where L_1 and L_2 are context-free languages
7. **R** **C** **DEC** **NONDEC**
   $L = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is finite}\}$
8. **R** **C** **DEC** **NONDEC**
   $L = L_1 - L_2$ where L_1 is context-free and L_2 is regular.
9. **R** **C** **DEC** **NONDEC**
   $L = L_1 \cap L_2$ where L_1 is regular and L_2 is an arbitrary language.

Problem 3: Short answer I (6 points)

(a) Give a regular expression for the set of all strings in $\{0, 1\}^*$ that contain at most one pair of consecutive 1's.

(b) Let M be a DFA. Sketch an algorithm for determining whether $L(M) = \Sigma^*$. Do not generate all strings (up to some bound on length) and feed them one-by-one to the DFA. Your algorithm must manipulate the DFA's state diagram.

Problem 4: Short answer II (5 points)

Let $\Sigma = \{a, b\}$, let $G = (V, \Sigma, R, S)$ be a CFG, and let $L = L(G)$. Give a grammar G' for the language $L' = \{wxw^R \mid w \in \Sigma^*, x \in L\}$ by modifying G appropriately.

Problem 5: Nonregularity (8 points)

Show that each of the languages below is not regular using the "direct argument" (you are advised to use the direct argument; however, if you wish to prove this in any other way, including the pumping lemma, you are welcome to do so). Assume $\Sigma = \{a, b\}$.

(a) $L = \{w \mid w \text{ is a palindrome}\}$.

(b) $L = \{w \mid w \text{ contains at least twice as many a's as b's}\}$.

Problem 6: TM design (6 points)

Give the state diagram of a TM M that does the following on input $\#w$ where $w \in \{0,1\}^*$. Let $n = |w|$. If n is even, then M converts $\#w$ to $\#0^n$. If n is odd, then M converts $\#w$ to $\#1^n$. Assume that ϵ is an even length string.

The TM should enter the accept state after the conversion. We don't care where you leave the head at the end of the conversion. The TM should enter the reject state if the input string is not in the right format. However, your state diagram does not need to explicitly show the reject state or the transitions into it.

Problem 7: Subset construction (10 points)

Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA that does not contain any epsilon transitions. The *subset construction* can be used to construct a DFA $M = \{Q', \Sigma, \delta', q_0', F')$ recognizing the same language as N. Fill in the following key details of this construction, using your best mathematical notation:

$Q' =$

$q_0' =$

$F' =$

Suppose that P is a state in Q' and a is a character in Σ. Then

$\delta'(P, a) =$

Suppose N has ϵ-transitions. How would your answer to the previous question change?

$\delta'(P, a) =$

Problem 8: Writing a proof (8 points)

We have seen that $ALL_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$ is undecidable.

(a) Show that $\overline{ALL_{CFG}}$ is Turing-recognizable.

(b) Is ALL_{CFG} Turing-recognizable? Explain why or why not.

Problem 9: RA modification (8 points)

Let L be a context-free language on the alphabet Σ. Let $(M, main, \{(Q_m, \Sigma \cup M, \delta_m, q_0^m, F_m)\}_{m \in M})$ be an RA recognizing L. Give an RA recognizing the language $L' = \{xy \mid x \in L \text{ and } y \in \Sigma^* \text{ and } |y| \text{ is even}\}$

(a) Explain the idea behind the construction.

(b) Give tuple notation for new RA.

Problem 10: Decidability (6 points)

Show that
$EQINT_{DFA} = \{\langle A, B, C\rangle \mid A, B, C$ are DFAs over the same alphabet Σ, and $L(A) = L(B) \cap L(C)\}$
is decidable.

This question does not require detail at the level of tuple notation. Rather, keep your proof short by exploiting theorems and constructions we've seen in class.

Problem 11: Reduction (8 points)

Let $\text{EVEN}_{TM} = \{\langle M \rangle \mid \text{M is a TM and } L(M) \text{ does not contain any string of odd length}\}$. Show that EVEN_{TM} is undecidable (you may assume A_{TM} — Turing machine membership — is undecidable).
You may *not* use Rice's Theorem.

Problem 12: Proof (8 points)

Prove that every binary string of even length can be derived from the following grammar:

$$S \to 0S0 \mid 1S1 \mid 1S0 \mid 0S1 \mid \epsilon$$