Complexity Homework 2

Released: February 21, 2008

Due: March 6, 2008

Problem 1:

A language A is *downward self-reducible* if there is a polynomial-time oracle machine M such that:

- $L(M^A) = A$. That is, when given an oracle for A, M decides A (self-reducibility).
- On input x, M only queries the oracle on strings *smaller* than x (downward reducibility).

The second restriction is necessary to make the property interesting – otherwise, on input x, M could just directly ask the oracle if $x \in A$.

(a) Show that SAT and TQBF are downward self-reducible.

(b) Show that if L is downward self-reducible, then $L \in \mathbf{PSPACE}$.

Problem 2:

An oracle machine is called a *robust oracle machine* if the language accepted by it remains the same no matter which oracle is used (however the running time may vary). Show that a language L is decided by M^K *in polynomial time* where M is a robust oracle machine and K is some oracle, if and only if $L \in \mathbf{NP} \cap \mathbf{co\text{-}NP}$.

Problem 3:

Show that $\mathbf{NP}^{\Sigma_k^p \cap \Pi_k^p} = \Sigma_k^p$ for all $k \geq 1$.

Problem 4:

We often assume that a circuit has NOT gates only at the input level.

(a) Show how to convert a circuit *with fan-out one for all gates* (except possibly for the input gates) into an equivalent circuit of no larger size, but with NOT gates only at the inputs.

(b) Show how to convert a general circuit (with any fan-out for the gates) into an equivalent circuit of at most twice the size, with NOT gates only at the inputs.

Problem 5 (Extra Credit):

Show that $\mathbf{NSPACE}(S) \subseteq \mathbf{ATIME}(S^2)$ for $S(n) \geq \log n$ (and S space-constructible).

(Hint: Recall the algorithm in the proof of Savitch's theorem. Use existential quantifiers to guess the "middle node" in paths and universal quantifier to check both halfs of the path exist. To be clear, give the certificate version of the ATM, clearly stating how many certificate tapes are there, how long each one is, and how the deterministic verification works and in what time.)

Conclude that $\mathbf{PSPACE} \subseteq \mathbf{AP}$. (Of course, we have already seen in class that $\mathbf{PSPACE} = \mathbf{AP}$.)

Problem 6 (Extra Credit):

What can you say about the class $\mathbf{\Sigma}_k^{p\,\mathbf{\Sigma}_\ell^p}$ for different values of k and ℓ? Prove your claim. (Remember that an oracle can be queried multiple times. Consider cases of k, ℓ being odd/even and 0, 1, 2 etc. To begin with, you may consider one or more of these special cases.)

Problem 7 (Extra Credit):

We have seen in class that PATH is $\mathbf{NL}$-complete with respect to log-space reductions. Define $\mathbf{NC}^1$ reduction and show that in fact PATH is $\mathbf{NL}$-complete with respect to $\mathbf{NC}^1$ reductions.