CS 273 Solutions for Quiz3

April 28, 2008

1 Question 1 (6 points)

Mark each of the following claims as true or false:

(a) The language $\{< M, w >| M$ is a TM and M halts on input $w\}$ is TM recognizable.

Solution:

True. It's recognizable but not decidable.

(b) The language $\{< M, w >| M$ is a TM and $L(M) = \emptyset\}$ is TM decidable

Solution:

False. E.g. from Rice's Theorem.

(c) The language $\{< M, w >| M$ is a DFA and $L(M) = \emptyset\}$ is TM decidable.

Solution:

True. You can decide it using a graph tracing algorithm

(d) The language $\{< G >| G$ is a CFG and $L(G) = \Sigma^*\}$ is TM decidable. (Σ is the alphabet given in the description of G.)

Solution:

False. We showed this in lecture 24.

(e) The language $\{< G >| G$ is a CFG and $L(G) = \emptyset\}$ is TM decidable.

Solution:

True. You can decide this using a graph tracing algorithm.

(f) There are more possible languages than there are different Turing machines.

Solution:

True. By diagonalization. See lecture 21.

(g) Every context-free language is decidable.

Solution:

True. See Lecture 20.

(h) A Turing machine can halt even if it hasn't read all of its input string.

Solution:

True. A TM halts when it enters the accept state, which it can do at any point it wants to.

2 Question 2 (4 points)

(a) Suppose that a Turing machine M contains the transition $\delta(q, 0) = (r, 2, R)$ (q and r are states; 0 and 2 are tape symbols.) If M is now in configuration $021q03$, what will its next configuration be?

(b) Suppose that a Turing machine M contains the transition $\delta(q, 0) = (r, 2, L)$ (q and r are states; 0 and 2 are tape symbols.) If M is now in configuration $021q03$, what will its next configuration be?

(c) Suppose that a Turing machine M contains the transition $\delta(r, 1) = (s, 2, L)$ (r and s are states; 1 and 2 are tape symbols.) If M is now in configuration $02r103$, what will its next configuration be?

Solution:

(a) $0212r3$, (b) $02r123$, (c) $0s2203$

3 Question 3 (3 points)

(a) What is the difference between "TM recognizable" and "TM decidable"?

Solution:

A decidable language is accepted by a TM that halts on all inputs. A recognizable language is accepted by a TM, but it might not halt on some input strings.

(b) An LBA is a Turing machine with one important restriction. What's the restriction?

Solution:

An LBA uses only the portion of the tape occupied by the input string. Or, equivalently, it uses only tape space whose length is proportional to the length of the input string.

(c) How does an enumerator differ from a normal Turing machine?

Solution:

An enumerator has an output tape rather than an input tape. Instead of deciding whether an input string is a member of the language, it writes all members of the language out onto its output tape.

4 Question 4 (4 points)

Briefly explain why the following statement is true. (Don't give a proof, just outline the key ideas.)

(a) $A_{\mathsf{TM}} = \{< M, w >| \ M$ is a TM and M accepts $w\}$ is TM recognizable but not TM decidable.

Solution:

We can recognize this language by simulating M on w, step by step. However, this simulation isn't guaranteed to halt: it will run forever if M never halts on w.

(b) The language $A_{\mathsf{CFG}} = \{< G, w >| \ G$ is a CFG and G generates $w\}$ is TM decidable.

Solution:

We can decide this language as follows. Convert G to Chomsky Normal form. A derivation for w in CNF has $2|w| - 1$ steps. (Ok if you didn't remember this equation exactly.) Generate all derivations of this length and check if any of them generates w.

Notice that A_{CFG} is a harder problem than deciding the language $L(G)$, which requires cooking up a TM that decides just this one language. Also some explanation is required if you want to have the TM "run" a PDA, because a TM is deterministic and a PDA isn't.

(c) If L and $\overline{L}$ are recognizable, then L is decidable.

Solution:

To build a decider for L, run the recognizers for L and $\overline{L}$ in parallel. If either one halts, halt and return the answer from L's decider or the negation of the answer from $\overline{L}$'s decider.

5 Question 5 (4 points)

Suppose that an extended TM is like a normal TM, except its transitions have the option of staying in place (S) rather than always having to move left or right. Show how to simulate the transition $\delta(s, a) = (r, b, S)$ using transitions of a normal TM. (Either write out the transitions as equations or draw a fragment of a state diagram.)

Solution:

To simulate this transition, we can first do $\delta(s, a) = (r', b, R)$ where r' is some new state name. Then we need a set of transitions $\delta(r', t) = (r, t, L)$ for each tape symbol t.
It's important that you have an extra new state, so you don't mess up the behavior of other transitions involving r and s. Another common mistake was to assume that the letter read in the section transition had to be b. This letter could be anything, since the read head is now one cell to the right of where the known b is.
The right way to read "any letter" is illustrated above. I took off 0.5 points for notations whose meaning was clear but which weren't actually correct notation for Turing machines.

6 Question 6 (4 points)

(a) Let M be a TM and w be a string. Define a TM M_w (with input alphabet Σ) as follows:

- Input $= x$
- If x is a palindrome, then accept.
- Otherwise, simulate M on w.
- Accept if M accepts. Reject if M rejects.

What are the possible values for $L(M_w)$?

Solution:

$L(M_w)$ is either Σ^* (if M accepts w) or the set of all palindromes in Σ^* (if M does not accept w)

(b) Let M be a TM and w be a string. Define a TM M_w (with input alphabet Σ) as follows:

- Input $= x$
- Simulate M on w.
- If M rejects, then reject.
- Otherwise (i.e. if M accepts), then accept if and only if x has even length.

What are the possible values for $L(M_w)$?

Solution:

$L(M_w)$ is either $\emptyset$ (if M does not accept w) or the set of all even length strings in Σ^* (if M accepts w).