CS 273: Intro to Theory of Computation, Spring 2008
Quiz 1 Solutions

Here is a literal solution to the 9/10am version of the quiz. After it, are solutions to variations of these questions from other versions of the quiz.

1. (2 points) To formally define a DFA, what five components do you need to specify?

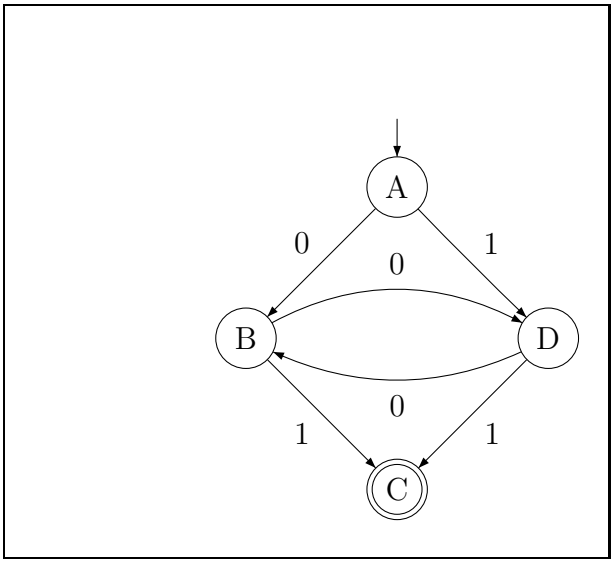
   Solution:
   The set of states, Q; input alphabet, Σ; transition function, δ; initial state, q_0; and set of final states, F.

2. (3 points) Suppose that $A = \{aa, bb\}$ and $B = \{1, 2\}$. List the members of $B \times \mathbb{P}(A)$.

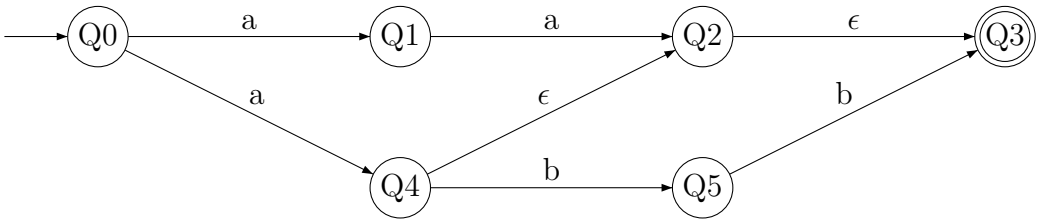
   Solution:
   $B \times \mathbb{P}(A) = \Big\{(1, \varnothing), (1, \{aa\}), (1, \{bb\}), (1, \{aa, bb\}), (2, \varnothing), (2, \{aa\}), (2, \{bb\}), (2, \{aa, bb\})\Big\}$

3. (2 points) Is the following a valid state diagram for a DFA? Explain your answer.

   Solution:
   No, because state C does not have output arrow for 0 (and also for 1).



4. (6 points) Here is the state diagram for an NFA.



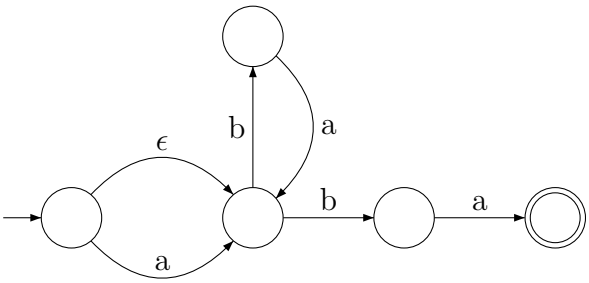
Suppose the transition function is named δ. Fill in the following output values for the transition function:

(a) $\delta(Q0, a) = \{\mathbf{Q1}, \mathbf{Q4}\}$

(b) $\delta(Q4, a) = \varnothing$

(c) $\delta(Q4, \epsilon) = \{\mathbf{Q2}\}$

5. (5 points) Give the state diagram of an NFA which recognizes the language represented by the regular expression $(a + \epsilon)(ba)^*ba$. (It's not necessary to follow any specific mechanical construction.)



6. (3 points) Give a regular expression for the following language

$$L = \{w \mid |w| \text{ is of even length}\}$$

when $\Sigma = \{a, b\}$.

Solution:
$(aa + ab + bb + ba)*$

7. (4 points) Let Σ be some alphabet. Suppose that $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ is a DFA recognizing L_1, $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ is a DFA recognizing L_2, and $N_3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$ is a DFA recognizing L_3. We can use the product construction to build a DFA M recognizing $(L_1 \cup L_2) - L_3$. Each state of M is a triple of states (q_1, q_2, q_3), where $q_1 \in Q_1$, $q_2 \in Q_2$, and $q_3 \in Q_3$.

(a) Precisely describe the set of final states for M.

Solution:

$$\big\{(q_1, q_2, q_3) : (q_1 \in F_1 \text{ or } q_2 \in F_2) \text{ and } q_3 \notin F_3\big\}$$

(b) Suppose that δ is the transition function for M. Give a formula for δ in terms of δ_1, δ_2, and δ_3. That is, if c is any character in Σ, then

$\delta((q_1, q_2, q_3), c) = \big(\boldsymbol{\delta(q_1, c), \delta(q_2, c), \delta(q_3, c)}\big)$

1 Other versions of questions 1 and 2

1. (2 points) True or false: for any regular language L, it's possible to create an NFA recognizing L which has only one final state.

   Solution: True. Create a new final state, with epsilon transitions from all the old final states.

2. (3 points) We've seen six operations that regular languages are closed under. Name five of them.

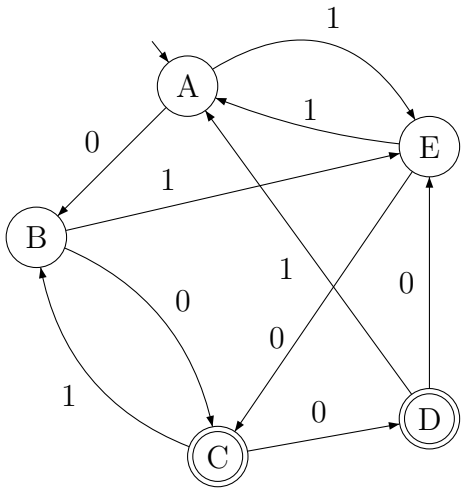
   Solution: actually, we've seen seven: union, intersection, start, concatenation, string reversal, homomorphism, set complement.

3. (3 points) In the context of regular languages, which of the following must always be finite

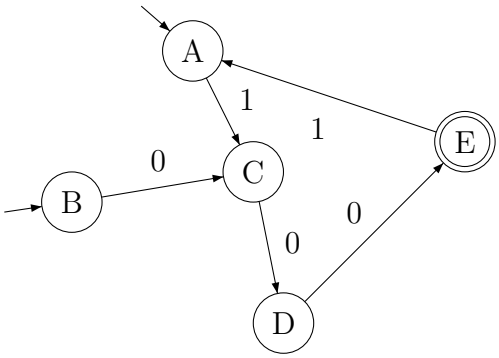
   - the length of a string **Solution:** yes
   - the number of strings in a language **Solution:** no
   - the number of states in a DFA **Solution:** yes

2 Question 3

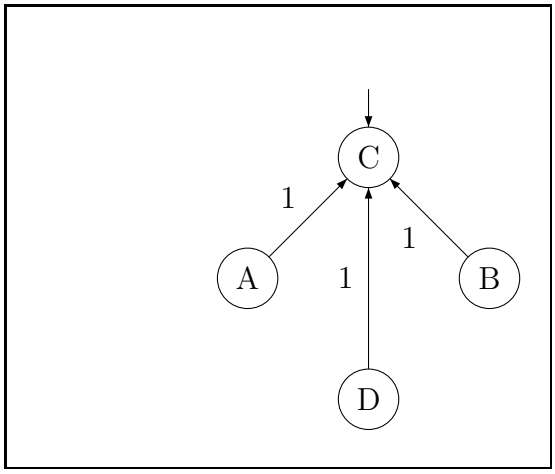
The following is a well-formed DFA diagram.



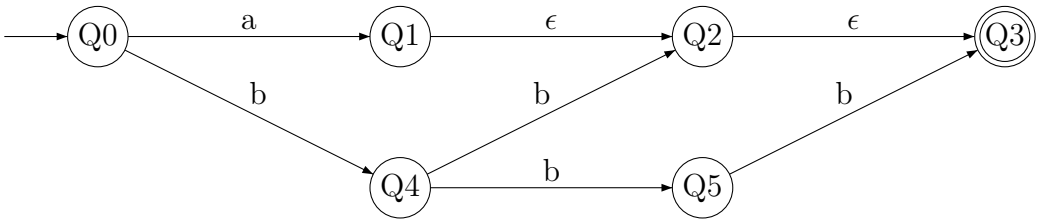
The following is not a legit NFA state diagram, because it has two start states.



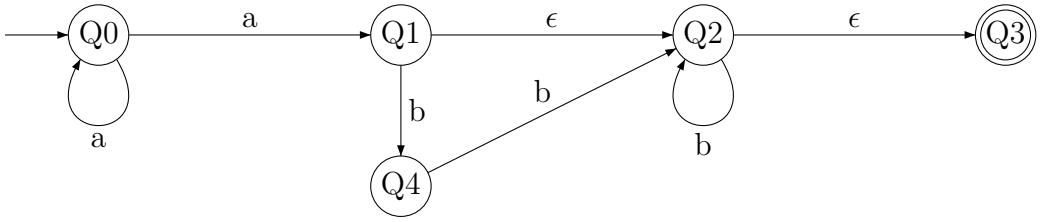
The following is a legal state diagram for an NFA, though it doesn't seem terribly useful in a practical sense.



3 Question 4



1. $\delta(Q0, a) = \{Q1\}$
2. $\delta(Q4, b) = \{Q2, Q5\}$
3. $\delta(Q4, \epsilon) = \emptyset$

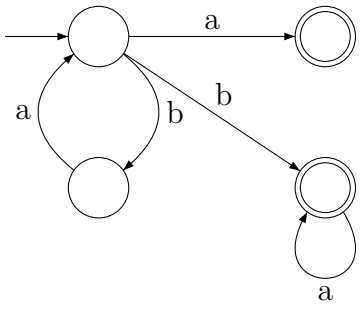


Suppose the transition function is named δ. Fill in the following output values for the transition function:

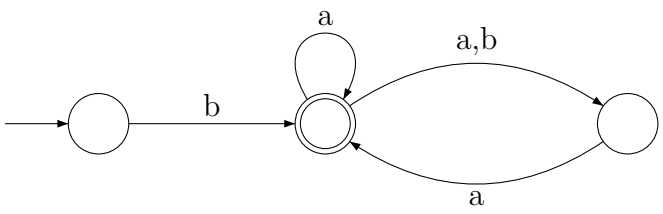
1. $\delta(Q0, a) = \{Q0, Q1\}$
2. $\delta(Q1, a) = \emptyset$
3. $\delta(Q2, b) = \{Q2\}$

4 Question 5

$(ba)^*(a + ba^*)$



$(ba^*)(aa + ba)^*$.



5 Question 6

odd length: $(a + b)[(a + b)(a + b)]^*$

2 mod 3: $(a + b)(a + b)[(a + b)(a + b)(a + b)]^*$

1 mod 4: $(a + b)[(a + b)(a + b)(a + b)(a + b)]^*$

6 Question 7

(4-points) Let Σ be some alphabet. Suppose that $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ is a DFA recognizing L_1 and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ is a DFA recognizing L_2. We can use the product construction to build a DFA M recognizing $(L_1 \cup L_2) - (L_1 \cap L_2)$. Each state of M is a pair of states (q_1, q_2), where $q_1 \in Q_1$ and $q_2 \in Q_2$.

(a) Precisely describe the set of final states for M.

Solution: $\{(q_1, q_2) \mid q_1 \in F_1 \text{ XOR } q_2 \in F_2\}$

where XOR is the exclusive OR function. That is, exactly one of the states in the pair is final.

(b) Suppose that δ is the transition function for M. Give a formula for δ in terms of δ_1 and δ_2. That is, if c is any character in Σ, then

$\delta((q_1, q_2), c) =$

Solution: $\delta((q_1, q_2), c) = (\delta_1(q_1, c), \delta_2(q_2, c))$