

CS 273: Intro to Theory of Computation, Fall 2007

Quiz 1 SOLUTIONS

1. (1 point) Is zero even?

Yes.

2. (2 points) Let $X = \{a, b\}$ and $Y = \{b, c\}$. List the elements of $\mathbb{P}(X) \cap \mathbb{P}(Y)$.

$\{\emptyset, \{b\}\}$

3. (5 points) Fill in the two missing parts of this definition:

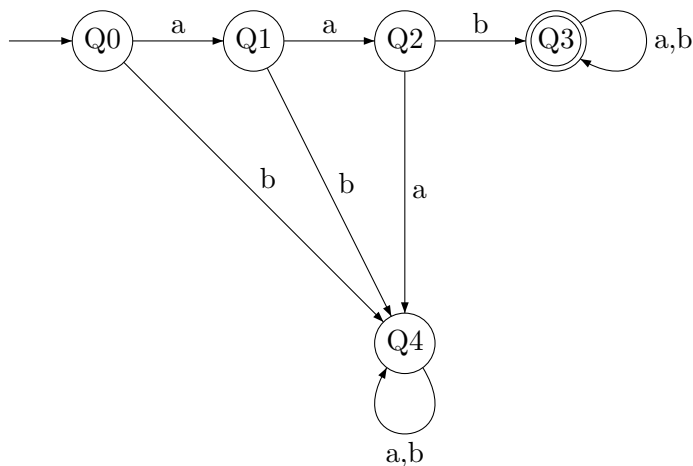
Suppose that $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA and w is a string. Let $w = w_1w_2\dots w_n$. Then M accepts w if there is a sequence of states $r_0r_1\dots r_n$ in Q such that

(a) $r_0 = q_0$

(b) $r_n \in F$

(c) $\delta(r_i, w_{i+1}) = r_{i+1}$ for every i between 0 and $n - 1$.

4. (4 points) Let $\Sigma = \{a, b\}$. Draw a state diagram for a DFA with alphabet Σ that accepts all strings starting in aab . (E.g. it accepts aab and $aabbba$, but not $abaab$.) Be sure to show all states and transitions.



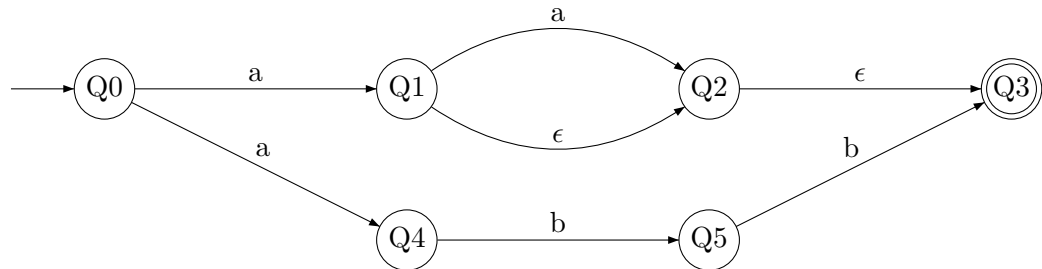
5. (5 points) To prove that regular languages are closed under union using only DFAs (not NFAs), we took two DFAs $M = (Q, \Sigma, \delta, q_0, F)$ and $N = (R, \Sigma, \gamma, r_0, G)$ and constructed a new DFA M' recognizing $L(M) \cup L(N)$. Each individual state of M' was a pair of states, one from Q and one from R .

(a) Name the set of all states in M' , using succinct mathematical notation. $Q \times R$

(b) Suppose that δ' is the transition function for M' . We needed to express the values of δ' in terms of the values given by δ and γ . Give the formula for $\delta'((q, r), c)$, where $q \in Q$, $r \in R$, and $c \in \Sigma$.

$$\delta'((q, r), c) = (\delta(q, c), \gamma(r, c))$$

6. (8 points) Here is the state diagram for an NFA.



Suppose the transition function is named δ . Fill in the following output values for the transition function:

(a) $\delta(Q0, a) = \{Q1, Q4\}$

(b) $\delta(Q4, b) = \{Q5\}$

(c) $\delta(Q1, \epsilon) = \{Q2\}$

(d) $\delta(Q1, b) = \emptyset$