CS173: Discrete Mathematical Structures, Spring 2008

First Midterm — February 14, 2008, 9:30am-10:45am

Name:	**Net ID:**

Circle your discussion section.

Time	Th 12:00-12:50	Th 1:00-1:50	Th 2:00-2:50	Fri 10:00-10:50	Fri 11:00-11:50
Discussion Section	1A Aparna A-H	2A Tracy A-Kr	3A Mo A-Me	4A Dan M. A-Lin	5A Dan M. A-N
	1B Dan S. I-Z	2B Nana Ku-Z	3B Lucas Mi-Z	4B Yuzi Liu-Z	5B Dan S. O-Z

<u>Instructions</u>

1. This is a closed-everything exam. No notes or electronics of any kind are allowed.
2. Print your full name and your NetID in the boxes above.
3. Print your name at the top of every page.
4. Please write clearly and legibly. If we can't read your answer, we can't give you credit.
5. You have 75 minutes to complete the exam, plan accordingly. Do not spend too much time on any one of the problems. Problems do not necessarily appear in order of difficulty.
6. You may write "I Don't Know" on any problem to receive 25% of the total credit. However, you must ONLY write "I Don't Know". If you write anything else, it will be graded as though it is a solution.

#	0	1-6	7	8	9	10	11	12	13	Total
Max	5	30	8	12	10	10	15	15	15	120
Score										

Course Questions

0. The easy problem:

   (a) [**1 point**] How many homeworks are you allowed to drop?

   (b) [**1 point**] What is the penalty for submitting late homework?

   (c) [**1 point**] What percentage of your course grade is devoted to discussion section participation?

   (d) [**2 points**] Circle the sources of information that you are ***not*** allowed to use while working on CS173 homework.

      i. The CS173 website.

      ii. Discussions with your friend Sarah who took CS173 last semester.

      iii. A textbook (other than the course text by Ensley and Crawley).

      iv. A brainstorming session with Josh, who is also taking CS173.

      v. A draft of the homework solutions that Josh produced after the brainstorming session.

      vi. Your section leader.

      vii. Wikipedia (a freely available online encyclopedia).

Multiple Choice (5 points each)

Indicate your answers by circling the correct one. **Each question has exactly one correct answer.** Read everything carefully.

1. What is the size of $\mathcal{P}(\mathcal{P}(\varnothing)) \times \mathcal{P}(\{1,2,3\})$? (Recall that we use $\mathcal{P}(A)$ to denote the powerset of a set A, and $\varnothing$ to denote the empty set.)

   (a) 64
   (b) 32
   (c) 16
   (d) 8
   (e) 6
   (f) 4
   (g) 2
   (h) 1
   (i) 0
   (j) None of the above.

2. *Read carefully.* Let $X = \{4, 8, 15, 16, 23, 42\}$. Which of the following is a partition of X?

   (a) $(\{4, 16\}, \{8, 15, 23\}, \{42\})$
   (b) $\{\{4, 16, 42, 4\}, \{15\}, \{8, 23\}\}$
   (c) $\{(4, 8, 16), (15, 23, 42)\}$
   (d) $\{4, 8, 15, 16, 23, 42\}$
   (e) None of the above.

3. Which of the following is the converse of the statement, "You are crazy if you are taking CS 173."

   (a) If you are crazy, then you are taking CS173.
   (b) If you are not taking CS 173, then you are not crazy.
   (c) If you are not crazy, then are you not taking CS 173.
   (d) You are taking CS173 and you are not crazy.
   (e) Butterflies are pretty.

4. Let A be the set of students and B be the set of classes that Albert takes. Consider the following predicates:

   - $P(x)$ means "x is stinky"
   - $Q(x)$ means "x has showered"
   - $R(x, y)$ means "x sits next to Albert in class y"

   Which of the following is equivalent to the statement "In every class Albert takes, some student that has not showered and is stinky sits next to Albert."

   (a) $\exists y \in B, \forall x \in A, R(x, y) \vee P(x)$
   (b) $\exists y \in B, \forall x \in A, R(y, x) \vee P(x)$
   (c) $\forall y \in B, \exists x \in A$, if $R(x, y)$ then $\neg Q(x)$
   (d) $\exists y \in B, \forall x \in A, R(x, y) \vee P(x) \vee \neg Q(x)$
   (e) $\exists y \in B, \forall x \in A, R(y, x) \vee P(x) \vee \neg Q(x)$
   (f) $\forall y \in B, \exists x \in A, R(y, x) \wedge P(x) \wedge \neg Q(x)$
   (g) $\forall y \in B, \exists x \in A, R(x, y) \wedge P(x) \wedge \neg Q(x)$
   (h) None of the above.

5. Which of the following is not a *member* of the set $\mathcal{P}(\{4, 5\}) \times \{1, 2, 3\} \times \{\pi, e\}$?

   (a) $(4, \{1, \pi\})$
   (b) $(\emptyset, 2, \pi)$
   (c) $(\{4, 5\}, 2, e)$
   (d) $(\{4\}, 3, \pi)$
   (e) None of the above.

6. Let $A(x)$ be the predicate "x is honest", and let $B(x)$ be the predicate "x is a politician". Let S be the set of people in the world. Which of the following is equivalent to the *negation* of the statement: $\forall x \in S$, if $\neg A(x)$ then $B(x)$?

   (a) All honest politicians are people.
   (b) Some dishonest person is not a politician.
   (c) Some honest person is not a politician.
   (d) Every politician is honest.
   (e) No politician is honest.
   (f) There is an honest politician.
   (g) There is a dishonest politician.
   (h) None of the above.

Short Answer

Write your solutions in the space provided.

7. **[8 points]** Fill in the blanks.

   (a) Every proof by induction is an argument that there can be no ________________________________ (2 words).

   (b) A proof by induction consists of three parts: the introduction (e.g. "The proof is by induction on n."), the ________________________ (2 words), and the ________________________ (2 words).

   (c) The ________________________ (2 words) allows us to assume that theorem holds for all smaller inputs than the one we are presently considering.

8. **[12 points]** Express the following sets as simply as you can. **Use explicit lists for finite sets.** For infinite sets, use set builder notation (both "form description" and "property description" answers are acceptable).

   Let $A = \{n \in \mathbb{Z} \ : \ n \text{ is even}\}$, $B = \{n \in \{2, 3, 4, \ldots\} \ : \ n \text{ is prime}\}$, $C = \{n^2 \ : \ n \in \mathbb{Z}\}$, and $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

   (a) $B \cap C$

   (b) $C - A$

   (c) $(B \cap D) \times (A \cap C \cap D)$

   (d) $\mathcal{P}(D \cap C \cap A) \cup \mathcal{P}((D \cap C) - A)$

9. **[10 points]** Count the number of integers in $\{0, 1, 2, \ldots, 500\}$ which are divisible by 4 or 5.

10. **[10 points]** Let $n \geq 1$ be an integer and let $U = \{1, 2, \ldots, n\}$. Prove or disprove:

    (a) $\forall (x, y) \in U \times U, \quad ((x - y) \in U) \vee ((y - x) \in U)$

    (b) $\forall A \in \mathcal{P}(U), \exists B \in \mathcal{P}(U), \quad (A \cup B = U) \wedge (A \cap B = \emptyset)$

Long Answer (15 points each)

Write your solutions in the space provided. (If you run out of room, you may continue your answer on another page, but please tell us where to look!)

11. Prove that for each integer $n \geq 0$, the following identity holds: $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$.

12. Let S be a subset of $\{1, 2, \ldots, 3n\}$ which contains $2n + 1$ numbers. Show that S contains 3 consecutive integers.

13. Prove by induction that any positive integer can be written as a sum of *distinct* powers of 2. 'Distinct' means that each power of 2 appears at most once in the sum. For example:

$$4 = 2^2 \qquad 17 = 2^4 + 2^0 \qquad 23 = 2^4 + 2^2 + 2^1 + 2^0 \qquad 173 = 2^7 + 2^5 + 2^3 + 2^2 + 2^0$$

In other words, prove that any positive integer can be written in binary!

(scratch paper)