CS 173: Midterm Exam 2

Spring 2006

Name: ____

NetID: ____

Lecture Section: 9:30a, 11:00a (circle one)

Section Leader: ___________________________

General Directions

1. Make sure your name is on every page.
2. Remember to write clearly and legibly. Unreadable answers will receive no credit.
3. This is a closed book exam. No notes of any kind are allowed. No calculators.
4. Remember to time yourself.

Question	Points	Out of
1		5
2		5
3		5
4		5
5		5
6		5
7		5
8		5
9		10
10		10
11		10
12		10
13		20
Total		100

Multiple Choice

Problem 1 (5pts)

What is n if $C(n, 3) = P(n, 2)$?

a) $\frac{19}{6}$

b) 8

c) There is no positive value of n that makes this equation true.

d) Some other positive value of n makes this equation true.

Problem 2 (5pts)

What is the coefficient of x^5 in $(3x + \frac{1}{2})^8$?

a) 21

b) $42\frac{1}{4}$

c) $354\frac{3}{8}$

d) 1701

Problem 3 (5pts)

W is a set of binary strings defined as follows: $10 \in W$, $01 \in W$, if $x \in W$ then $1x0 \in W$, and if $x \in W$ then $0x1 \in W$. Nothing else is in W. Let $S = \{x | x \in W, |x| = n\}$. That is, S is the set of strings in W that have length n. How many strings are in S?

a) $\binom{n}{\frac{n}{2}}$

b) $2^{\frac{n}{2}}$

c) 2^{n-1}

d) $2\binom{n}{\frac{n}{2}}$

Problem 4 (5pts)

Suppose a password must be exactly six characters long, where each character is an element of the set

$$\{A, B, C, D, E, F, G, H, I, J, 1, 2, 3, 4\}.$$

Furthermore, suppose each password must contain at least one letter. How many such passwords are there? (Duplicate characters are allowed.)

a) $10 \cdot \binom{14}{5}$

b) $10 \cdot \frac{14!}{10!}$

c) $14^6 - 4^6$

d) $10 \cdot 4^5 \cdot 6!$

Problem 5 (5pts)

A box contains 10 French books, 20 Spanish books, 8 German books, 15 Russian books, and 25 Italian books. Suppose we randomly select books from boxes so that each book is equally likely to be chosen. How many books must we choose from the boxes in order to be sure of choosing 12 books of the same language?

a) 52

b) 56

c) 57

d) 60

Problem 6 (5pts)

A sushi bar sells five kinds of rolls, and you want to eat nine rolls for dinner. How many different sushi dinners do you have to choose from if you must have one roll of each type? (Note, the arrangement of the sushi on your plate is irrelevant.)

a) 5^4

b) 5^9

c) $\frac{9!}{4!}$

d) $\binom{8}{4}$

Problem 7 (5pts)

At a party, each card in a standard deck of 52 unique cards is torn in half and both halves are placed in a box. Two guests each draw a half-card from the box. What is the probability they draw 2 halves of the same card?

a) $\frac{1}{52}$

b) $\frac{1}{206}$

c) $\frac{1}{103}$

d) $\frac{52}{103}$

Problem 8 (5pts)

A directory on a computer's hard drive contains 12 files, 3 of which are corrupted by a virus. If a corrupted file is selected, the virus is removed and another file is selected at random from the 12 files. That is, a file may be accessed more than once, but after the first time, it is certainly not corrupt.

Find the probability that exactly three selections are made until an uncorrupted file is chosen. In other words, find the probability the first two selections are corrupted, and the third is not.

a) 1

b) $\frac{11}{288}$

c) $\frac{3}{64}$

d) 4

Short Answer Problems

Problem 9 (10pts)

Give a combinatorial proof that $\binom{n}{k+1} = \frac{(n-k)}{(k+1)}\binom{n}{k}$. You may use the template below for your response, or you may write your own.

Fill in the blanks for the proof:

The left side of the equation counts the number of different sets of size ________ that can be selected from a set of size ________.

The right side of the equation counts the same thing: for any of the ________ ways of selecting k elements from a set of size n, there are ________ ways of selecting one additional element, thereby making a total of ________ elements selected from n, for a total of ________ sets. (Draw a picture illustrating this situation below.) Unfortunately, this over-counts by a factor of ________. To see that this is true, consider any single set of size $k+1$. It can be partitioned into a set of size ________ and a set of size one in ________ different ways, each of which we counted separately. To rectify this over-counting, we simply divide by ________. Thus we have a total of $\frac{(n-k)}{(k+1)}\binom{n}{k}$ different ways of selecting $k+1$ items from n.

Problem 10 (10pts)

Define a recursive function $listmax()$ whose input is a finite set of integers $L_n = \{x_1, x_2, \ldots, x_n\}$ with $n \geq 2$, and whose output is the maximum element in the set. You may assume the existence of a function $max(\{a, b\})$ whose input is a list of length 2, and whose output is the maximum element of the list.

Problem 11 (10pts)

Consider the Fibonacci Sequence: $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$, for $n \geq 2$. In this problem you will prove that $f_n > A \cdot (\frac{3}{2})^n$ when $n \geq 1$, for some constant A.

a) Find a value of A that makes the expression true.

b) As the base case of an inductive argument, prove that $f_1 > A \cdot \frac{3}{2}$ and $f_2 > A \cdot (\frac{3}{2})^2$, where A is the value you chose in part a). (Think about why both these base cases are necessary.)

c) Complete the proof by induction.

Problem 12 (10pts)

In this problem we will find the number of 3 element subsets of the set $\{1, 2, \ldots, 10\}$ whose sums are odd. The set $\{1, 2, 4\}$ is an example of such a set, but $\{1, 2, 3, 5\}$ and $\{2, 3, 5\}$ are not.

a) How many ways are there of choosing 2 odd numbers from the set $\{1, 2, \ldots, 10\}$?

b) How many ways are there of choosing 2 odd numbers and 4 even numbers from the set $\{1, 2, \ldots, 10\}$?

c) Describe all the different ways, in terms of even and odd numbers, that a three element set can have an odd total. By way of getting you started, can the sum be odd if all three elements are even?

d) Use your answer from part c) and the expertise you gained in parts a) and b) to find the number of 3 element subsets of the set $\{1, 2, \ldots, 10\}$ whose sums are odd.

Problem 13 (20pts)

You are most likely getting hungry by now, so we're going to count sandwiches. Suppose the local Freeway Sandwich Shop has 6 different kinds of meat, 3 different kinds of cheese, 12 different kinds of vegetable, and 2 different kinds of bread. Note that because of our discriminating tastes, sandwiches whose ingredients are stacked in different orders are considered to be different sandwiches, even if the ingredient list is the same. Moreover, the sandwich makers at Freeway are very good–they always remember to put the bread on the outside of the sandwich. DON'T FORGET THESE FACTS!

a) Suppose you want a cheese (only) sandwich on white bread. How many different sandwiches do you have to choose from if you want 6 slices of cheese? (Duplicate cheese slice types are allowed.)

b) If you randomly choose sandwiches like those in part a), how many different sandwiches would you have to choose in order to assure that you choose at least 3 that are exactly the same?

c) How many different sandwiches can be made from 3 slices of turkey, 2 slices of cheddar cheese, and one onion (a vegetable)? (Don't forget to think about the bread!)

d) If you prefer that your sandwiches be stacked in order–meats first, then cheeses, then veggies–how many italian-bread sandwiches consist of a stack of 3 different meats, followed by 2 different cheeses, followed by one veggie?

e) How many of the sandwiches you counted in part d) have stacks that begin with turkey and roast beef?

f) What is the probability of selecting a sandwich like those in part e) if you choose one randomly from those in part d)?

SCRATCH PAPER