CS 173: Midterm Exam 2

Fall 2005

Name: _____

NetID: _____

Lecture Section: _____

Section Leader: _____

General Directions

1. Make sure your name is on every page.
2. Remember to write clearly and legibly. Unreadable answers will receive no credit.
3. This is a closed book exam. No notes of any kind are allowed. No calculators.
4. Remember to time yourself.

Question	Points	Out of
1		5
2		5
3		5
4		5
5		5
6		5
7		5
8		5
9		10
10		10
11		10
12		10
13		20
Total		100

Multiple Choice

Problem 1 (5pts)

Which one of the following is not a part of a proof by induction?

a) Show $P(b)$ holds for some b in the natural numbers.

b) Assume $P(n)$ holds for some $n \geq b$.

c) Assume $P(n) \longrightarrow P(n+1)$ whenever $n \geq b$.

d) Conclude that for all $n \geq b$, $P(n)$ is true.

e) All of these appear in a complete proof.

Problem 2 (5pts)

Which of the following best describes the worst case running time of the following algorithm if "twiddle-thumbs" runs in time $O(n)$?

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Input:  n = 2^k for some integer k.

i = n
while i ≥ 1
      for j = 1 to n
           twiddle-thumbs
      end for
      i = i/2
end while
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a) $O(\log n)$

b) $O(n)$

c) $O(n \log n)$

d) $O(n^2 \log n)$

e) $O(2^n)$

Problem 3 (5pts)

W is a set of integers defined as follows: $1 \in W$, $2 \in W$, if $x \in W$ then $(3 + x) \in W$. Nothing else is in W. Which of the following best describes W?

a) positive integers evenly divisible by 3

b) positive integers

c) positive integers not evenly divisible by 3

d) primes and square numbers

e) None of these describe W.

Problem 4 (5pts)

Suppose a white bucket contains 8 blue marbles and 4 orange marbles. We draw 2 marbles from the bucket without replacing them. If we assume that all the marbles are equally likely to be chosen, what is the probability that either both marbles drawn are blue or the first one is orange?

a) $7/9$

b) $3/4$

c) $4/5$

d) $25/33$

e) None of the above.

Problem 5 (5pts)

Which of the following sets is countably infinite?

a) The even integers greater than -5 and less than 5.

b) The set of circles in the plane.

c) The set of real numbers that are not rational.

d) The set of functions whose domain is the set of natural numbers, and whose co-domain is $\{0, 1, 2, \ldots, 9\}$.

e) None of the above.

Problem 6 (5pts)

How many routes are there from the lower left corner of an $n \times n$ square grid to the upper right corner if we are restricted to traveling only to the right or upward?

a) $\binom{2n}{n}$

b) n^2

c) $2n$

d) $n!$

e) None of the above.

Problem 7 (5pts)

What is the coefficient of x^6y^5 in the expansion of $(2x^2 + 3y)^8$?

a) $\binom{8}{3}2^3 3^5$

b) $\binom{11}{6}$

c) $\binom{11}{6}2^3 3^5$

d) $\binom{8}{6}2^6 3^5$

e) None of the above.

Problem 8 (5pts)

In how many ways can 4 married couples be seated in a row of 8 seats if each person must sit by his or her spouse?

a) $\frac{8!}{2}$

b) $4! \cdot 2^4$

c) $4!$

d) $8! \cdot 2^4$

e) None of the above.

Short Answer Problems

Problem 9 (10pts)

Label each of the following sets as finite, countably infinite, or uncountable. Justify your responses briefly for partial credit.

a) The set of all correct Java computer programs.

b) The set of all computer programs in Java that have ever been written.

c) The set of rational numbers between 1 and 2.

d) The set of functions with domain $\{0, 1, 2\}$ and codomain $N = \{1, 2, 3, \ldots\}$.

e) $\{x : x$ is the square root of a positive rational number$\}$.

Problem 10 (10pts)

At any party with more than 7 guests, the entire group can be divided into conversations between 3 or 5 people. For example, if there are 13 guests, the group can be divided into 2 five person conversations, and 1 three person conversation.

a) Convince yourself that the statement above is true by describing the conversation groups for parties of size 8, 22, and 187.

b) Use induction to prove that the statement is true. Think carefully about your base case(s).

Problem 11 (10pts)

Give a combinatorial proof that $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$. That is, prove the equation by showing that the left hand side and the right hand side count the same set of possibilities.

Problem 12 (10pts)

Prove that if 65 pins are stuck into an 8 inch square bulletin board, there are at least one pair of pins closer than $\sqrt{2}$ inches apart.

Problem 13 (20pts)

a) Suppose there are 8 players on the Illinois tennis team, and 10 players on the Michigan tennis team. How many different Illinois vs. Michigan singles tennis matches could be arranged?

b) How many different doubles teams could be chosen from among the Illinois players?

c) How many different Illinois vs. Michigan doubles matches could be arranged?

d) How many different gift boxes of 12 wedges of cheese can be made using wedges of Cheddar, Edam, Gouda, and Swiss cheese?

e) How many different gift boxes of 12 wedges of cheese can be made using wedges of Cheddar, Edam, Gouda, and Swiss, if 2 of each kind must be included?

f) There are 30 students in a campus organization. How many different groups of students of size 3 or larger can be selected from among the 30? Just give a succinct expression for the number. You don't have to compute it.

g) How many different ways are there to arrange the letters in the word illinois?

h) If a 9 member committee is selected at random from among 10 faculty and 20 students, what is the probability that it contains exactly 3 faculty and 6 students?

SCRATCH PAPER