CS173 Cheat Sheet (Spring 2008)

Set Theory Notation		
empty set	$\varnothing$	$\{\ \}$
subset	$A \subseteq B$	$\forall x\colon x \in A \rightarrow x \in B$
proper subset	$A \subset B$	$A \subseteq B \wedge \exists y \in B\colon y \notin A$
superset	$A \supseteq B$	$B \subseteq A$
proper superset	$A \supset B$	$B \subset A$
set equality	$A = B$	$A \subseteq B \wedge B \subseteq A$
union	$A \cup B$	$\{x \mid x \in A \vee x \in B\}$
intersection	$A \cap B$	$\{x \mid x \in A \wedge x \in B\}$
difference	$A - B$	$\{x \mid x \in A \wedge x \notin B\} = A \cap \overline{B}$
symmetric difference	$A \Delta B$	$\{x \mid x \in A \leftrightarrow x \notin B\}$
complement	$\overline{A}$	$\{x \mid x \notin A\} = U - A$
Cartesian product	$A \times B$	$\{(a,b) \mid a \in A \wedge b \in B\}$
power set	$\mathcal{P}(A)$	$\{B \mid B \subseteq A\}$
cardinality	$\lvert A \rvert$	# of elements (if finite)

Binary relation $\boldsymbol{R \subseteq A \times A}$	
relation notation	a and b are related $\iff (a,b) \in R$
inverse R^{-1}	$\{(b,a) \in A \times A \mid (a,b) \in R\}$
reflexive	$\forall a \in A,\ (a,a) \in R$
symmetric	$\forall a,b \in A$, if $(a,b) \in R$ then $(b,a) \in R$
antisymmetric	$\forall a,b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a = b$
transitive	$\forall a,b,c \in A$, if $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$

Equivalence relation $\boldsymbol{\sim}$

An equivalence relation is a binary relation which is reflexive, symmetric, and transitive

Partial order $\boldsymbol{\preceq}$

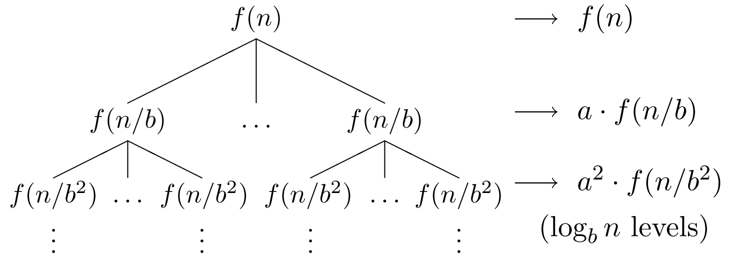
A partial order, or poset, is a binary relation which is reflexive, antisymmetric, and transitive.

Function $\boldsymbol{f\colon A \rightarrow B}$

A function f from A to B associates each element $a \in A$ to exactly one element $b \in B$.

Notation	$b = f(a)$ if b is associated to a
one-to-one (or injective)	$\forall a_1, a_2 \in A$, if $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$
onto (or surjective)	$\forall b \in B$, $\exists a \in A$ such that $f(a) = b$
bijection	one-to-one *and* onto
inverse $f^{-1}\colon B \rightarrow A$	$\{(b,a) \mid b = f(a)\}$ (if f is a bijection)

Recursion tree for $T(n) = aT(n/b) + f(n)$



$$\exists c < 1\colon a \cdot f(n/b) = c \cdot f(n) \implies T(n) = \Theta(f(n))$$

$$a \cdot f(n/b) = f(n) \implies T(n) = \Theta(f(n) \log n)$$

$$\exists c > 1\colon a \cdot f(n/b) = c \cdot f(n) \implies T(n) = \Theta(n^{\log_b a})$$

Asymptotic notation

$f(n) = o(g(n))$	$\forall c > 0\colon \exists N > 0\colon \forall n \geq N\colon f(n) < c \cdot g(n)$
$f(n) = O(g(n))$	$\exists c > 0\colon \exists N > 0\colon \forall n \geq N\colon f(n) \leq c \cdot g(n)$
$f(n) = \Theta(g(n))$	$f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$
$f(n) = \Omega(g(n))$	$\exists c > 0\colon \exists N > 0\colon \forall n \geq N\colon f(n) \geq c \cdot g(n)$
$f(n) = \omega(g(n))$	$\forall c > 0\colon \exists N > 0\colon \forall n \geq N\colon f(n) > c \cdot g(n)$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \begin{cases} = 0 & \implies f(n) = o(g(n)) \\ < \infty & \implies f(n) = O(g(n)) \\ > 0 & \implies f(n) = \Omega(g(n)) \\ = \infty & \implies f(n) = \omega(g(n)) \end{cases}$$

$$f(n) = o(g(n)) \iff g(n) = \omega(f(n))$$

$$f(n) = O(g(n)) \iff g(n) = \Omega(f(n))$$

$$f(n) = \Theta(g(n)) \iff g(n) = \Theta(f(n))$$

$$f(n) = O(g(n)) \implies f(n) + h(n) = O(g(n) + h(n))$$

$$f(n) = O(g(n)) \implies f(n) \cdot h(n) = O(g(n) \cdot h(n))$$

$$f(n) + g(n) = O(\max\{f(n), g(n)\})$$

$$f(n) = O(g(n)) \text{ and } g(n) = O(h(n)) \implies f(n) = O(h(n))$$

$$\sum_{i=0}^{\infty} \alpha = \frac{1}{1-\alpha} \quad (\text{if } \alpha < 1)$$

$$\sum_{i=0}^{d} i^c = \Theta(n^{c+1}) \quad (\text{if } c \neq -1)$$

$$\sum_{i=0}^{n} c^i = \Theta(c^n) \quad (\text{if } c > 1)$$

$$\sum_{i=1}^{n} \log i = \Theta(n \log n)$$

Logarithm identities

$$\log_b(b^x) = x$$

$$b^{\log_b x} = x$$

$$\log_b x = \frac{\log_c x}{\log_c b}$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b(1/x) = -\log_b x$$

$$x^{\log_b y} = y^{\log_b x}$$

$$\log_b(x^y) = y \log_b x$$