

# Complexity Test 1

February 22, 2007

This is a closed-book, closed-notes, individual-effort test. Each problem is worth the same amount, but not all are of equal difficulty. Please write your name on each sheet of your solutions.

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## Problem 1:

In class we saw a theorem that stated  $\mathbf{DSPACE}(S) \subseteq \mathbf{DTIME}(O(2^S))$ .

- Nevertheless, show that  $\mathbf{DSPACE}(O(1)) \not\subseteq \mathbf{DTIME}(O(1))$ .
- Why is the above theorem not applicable here?

## Problem 2:

Show that **co-NP** is closed under intersection.

## Problem 3:

Show that  $\mathbf{NP}^{\Sigma_k^p \cap \Pi_k^p} = \Sigma_k^p$  for all  $k \geq 1$ .

## Problem 4:

A *strong non-deterministic TM* is a Turing machine that makes nondeterministic choices, and has three kinds of terminating states: *yes*, *no*, and *maybe*. We define its accepting condition as follows: A language  $L$  is decided by a strong NTM if:

- on input  $x \in L$ , one or more threads terminate at the *yes* state and all other threads terminate in the *maybe* state (none terminate in the *no* state).
- on input  $x \notin L$ , one or more threads terminate at the *no* state and all other threads terminate in the *maybe* state (none terminate in the *yes* state).

Show that a language  $L$  is decided by a strong NTM in polynomial time if and only if  $L \in \mathbf{NP} \cap \mathbf{co-NP}$ .

## Problem 5:

Show that the following problem is in **NP** if and only if  $\mathbf{co-NP} = \mathbf{NP}$ :

$$\text{MAXSAT} = \left\{ (\phi, k) \mid \begin{array}{l} \phi \text{ is a 3SAT formula and } k \text{ is (exactly) the greatest} \\ \text{number of clauses simultaneously satisfiable in } \phi \end{array} \right\}$$