Complexity Homework 1 (first half)

Released: January 23, 2007

Due: February 1, 2007

Some of these problems are from chapter 2 in the Arora-Barak (AB) text.

For problems that involve nondeterministic time classes, sometimes the solution is simpler when phrased in terms of nondeterministic Turing machines, and other times simpler when phrased in terms of "certificates".

Problem 1 [AB #3]:

Show that the halting problem is **NP**-hard. Is it **NP**-complete?

Problem 2:

(a) [AB #5] Let L_1, L_2 be languages in **NP**. Are $L_1 \cup L_2$ and $L_1 \cap L_2$ necessarily in **NP**?

(b) Let L_1, L_2 be languages in **NP**. Show that $L_1 L_2$ and L_1^* are in **NP**.

(c) Let L_1, L_2 be languages in **P**. Show that $L_1 L_2$ and L_1^* are in **P**.

Problem 3 [AB #21]:

Let L_1, L_2 be languages in $\mathbf{NP} \cap \textbf{co-NP}$. Show that their symmetric difference

$$L_1 \oplus L_2 \stackrel{\text{def}}{=} \{x \mid x \text{ is in exactly one of } L_1, L_2\}$$

is also in $\mathbf{NP} \cap \textbf{co-NP}$.

Problem 4:

Show that 2SAT is in **P**.

Hint: Consider a directed graph with all the literals as nodes, and edges as implications ($(x \vee y)$ corresponds to $(\neg x \Rightarrow y)$ and $(\neg y \Rightarrow x)$). Look to derive contradictions of the form $(\neg x \Rightarrow x)$ and $(x \Rightarrow \neg x)$. What do such contradictions tell you about a possible satisfying assignment?

Problem 5:

Show that the following are equivalent [(a) $\Rightarrow$ (b) is almost AB #23]:

(a) Every unary[1] language in **NP** is also in **P**.

(b) $\mathbf{DTIME}(2^{O(n)}) = \mathbf{NTIME}(2^{O(n)})$ (these are often called **E** and **NE**, respectively).

Hint: It takes $\Theta(\log n)$ bits to encode the number "n" in binary.

Problem 6 (Extra credit) [AB #13]:

Show that if there is a unary language that is **NP**-complete, then $\mathbf{P} = \mathbf{NP}$.

[1] A language is *unary* if it is a subset of $\{1\}^*$ — that is, it only uses one symbol of the alphabet.