

# CS 473G: Graduate Algorithms, Spring 2007

## Homework 1

Due February 6, 2007

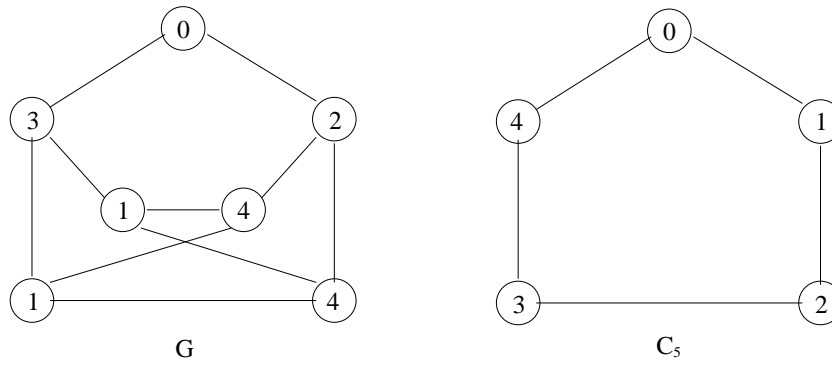
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Remember to submit **separate, individually stapled** solutions to each of the problems.

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1. Jeff tries to make his students happy. At the beginning of class, he passes out a questionnaire to students which lists a number of possible course policies in areas where he is flexible. Every student is asked to respond to each possible course policy with one of “strongly favor”, “mostly neutral”, or “strongly oppose”. Each student may respond with “strongly favor” or “strongly oppose” to at most five questions. Because Jeff’s students are very understanding, each student is happy if he or she prevails in just one of his or her strong policy preferences. Either describe a polynomial time algorithm for setting course policy to maximize the number of happy students or show that the problem is NP-hard.
2. Consider a variant 3SAT’ of 3SAT which asks, given a formula  $\phi$  in conjunctive normal form in which each clause contains at most 3 literals and each variable appears in at most 3 clauses, is  $\phi$  satisfiable? Prove that 3SAT’ is NP-complete.
3. For each problem below, either describe a polynomial time algorithm to solve the problem or prove that the problem is NP-complete.
  - (a) A *double-Eulerian* circuit in an undirected graph  $G$  is a closed walk that traverses every edge in  $G$  exactly twice. Given a graph  $G$ , does  $G$  have a double-Eulerian circuit?
  - (b) A *double-Hamiltonian* circuit in an undirected graph  $G$  is a closed walk that visits every vertex in  $G$  exactly twice. Given a graph  $G$ , does  $G$  have a double-Hamiltonian circuit?
4. Suppose you have access to a magic black box; if you give it a graph  $G$  as input, the black box will tell you, in constant time, if there is a proper 3-coloring of  $G$ . Describe a polynomial time algorithm which, given a graph  $G$  that is 3-colorable, uses the black box to compute a 3-coloring of  $G$ .
5. Let  $C_5$  be the graph which is a cycle on five vertices. A  $(5, 2)$ -coloring of a graph  $G$  is a function  $f : V(G) \rightarrow V(C_5)$  which maps the vertices of  $G$  to the vertices of  $C_5$  so that every pair  $\{u, v\}$  of adjacent vertices in  $G$  is mapped to a pair  $\{f(u), f(v)\}$  of vertices in  $C_5$  which are at distance two from each other.

For example, the graph  $G$  below has a  $(5, 2)$ -coloring; a vertex  $u$  in  $G$  is mapped to the vertex in  $C_5$  according to the label on  $u$ .



Using a reduction from 5COLOR, prove that the problem of deciding whether a given graph  $G$  has a  $(5, 2)$ -coloring is NP-complete.