1b

$$L = \{x_1 \# x_2 \# \cdots x_k \mid k \geq 2, \ x_i = x_j^R \text{ for some } i \neq j\}$$

d

5

$$\Sigma = \{a, b, \#\}$$

$$L = \{x \# y^R \mid x, y \in \{a, b\}^*, x \neq y\}$$

To Show:

For every CFG $G$ there is a PDA $M$ s.t.

$$L(G) = L(M)$$

$$G = (V, \Sigma, R, S)$$

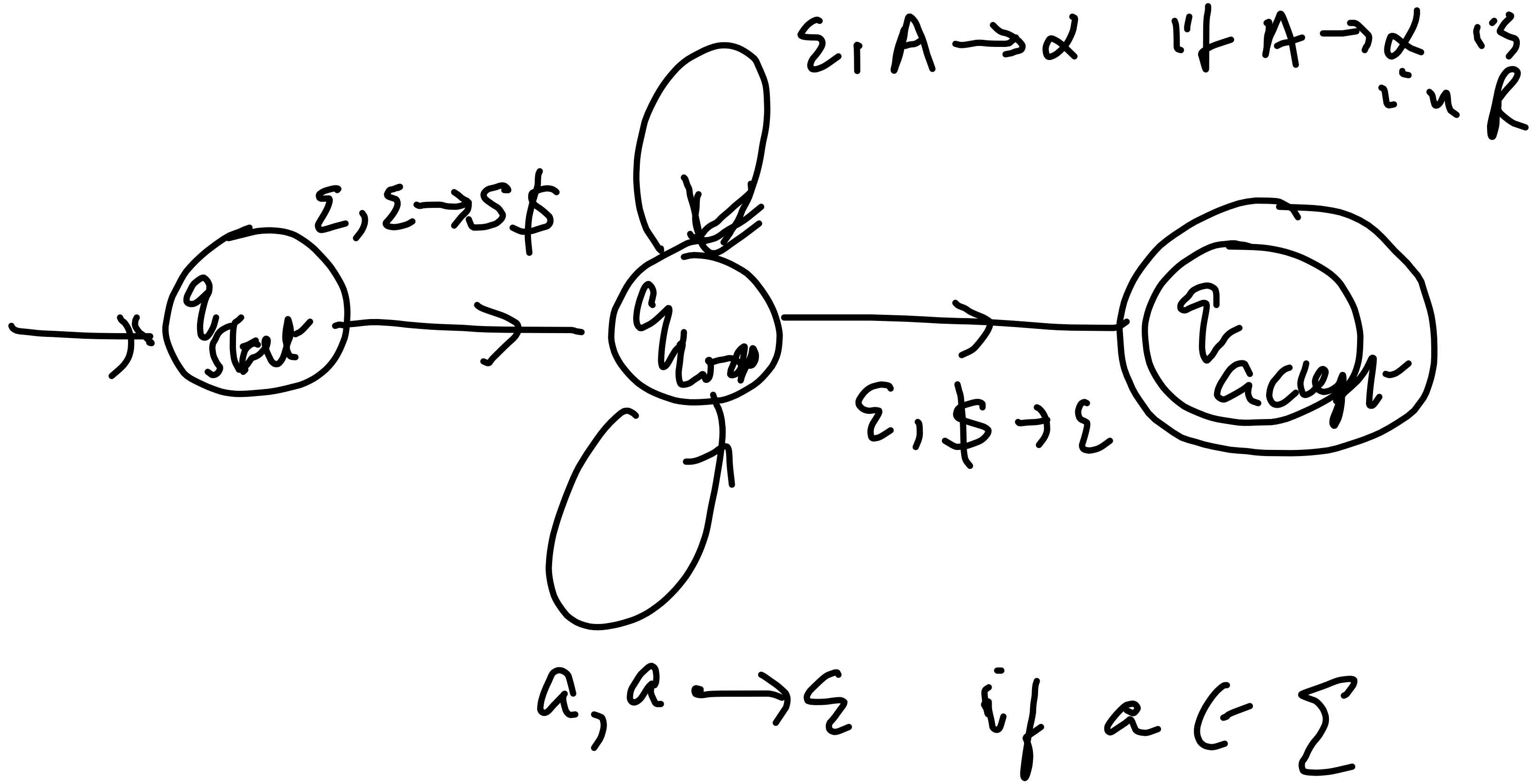
$$M = (Q, \Sigma, \Gamma, \delta, q_0, F)$$

$$Q = \{q_{start}, q_{loop}, q_{accept}\}$$

$$\Gamma = V \cup \Sigma \cup \{\$\}$$

$$q_0 = q_{start}$$

$$F = \{q_{accept}\}$$

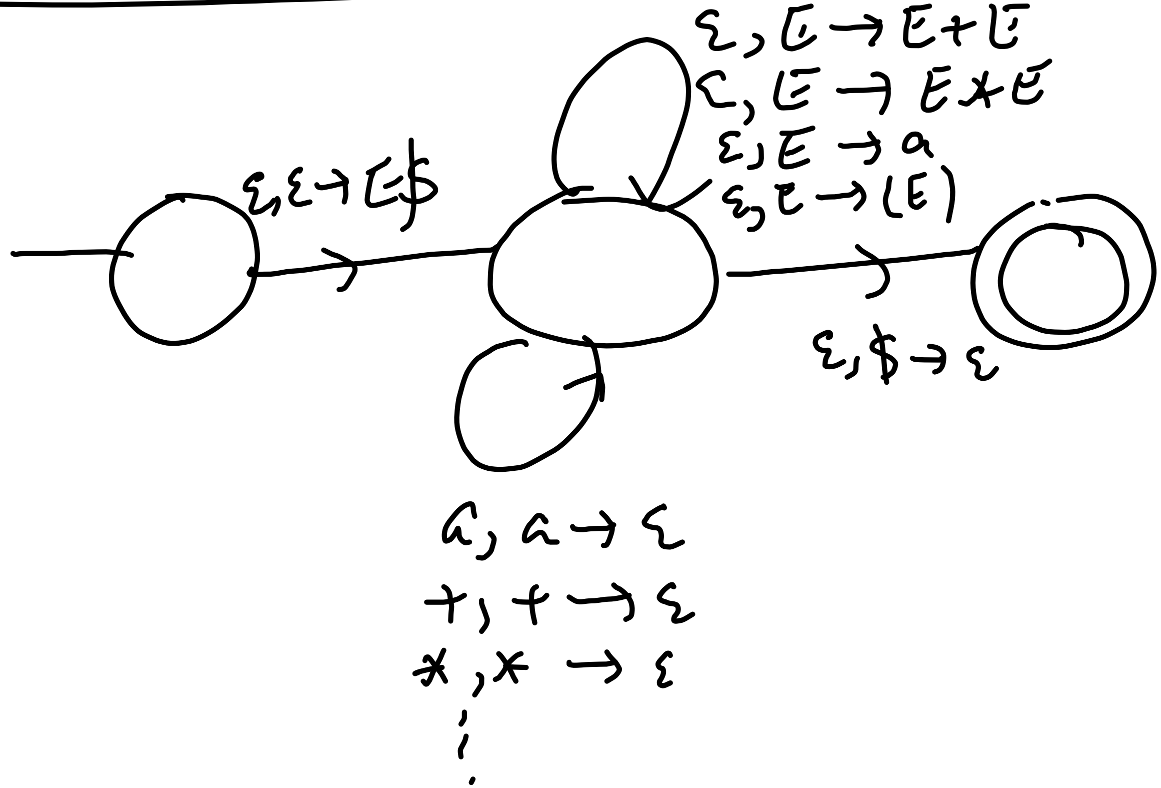

$q_{start}$
$\varepsilon, \varepsilon \to S\$$
$q_{loop}$
$\varepsilon, A \to \alpha$ if $A \to \alpha$ is in $R$
$a, a \to \varepsilon$ if $a \in \Sigma$
$\varepsilon, \$ \to \varepsilon$
$q_{accept}$

$E \to E * E \mid E + E \mid (E) \mid a$

$G = (V, \Sigma, R, S)$ $\quad \Sigma = \{a, +, *, (, )\}$

$S = E$ $\quad V = \{E\}$

---



Why does this work.

$w \in L(G)$ then $M$ should accept.

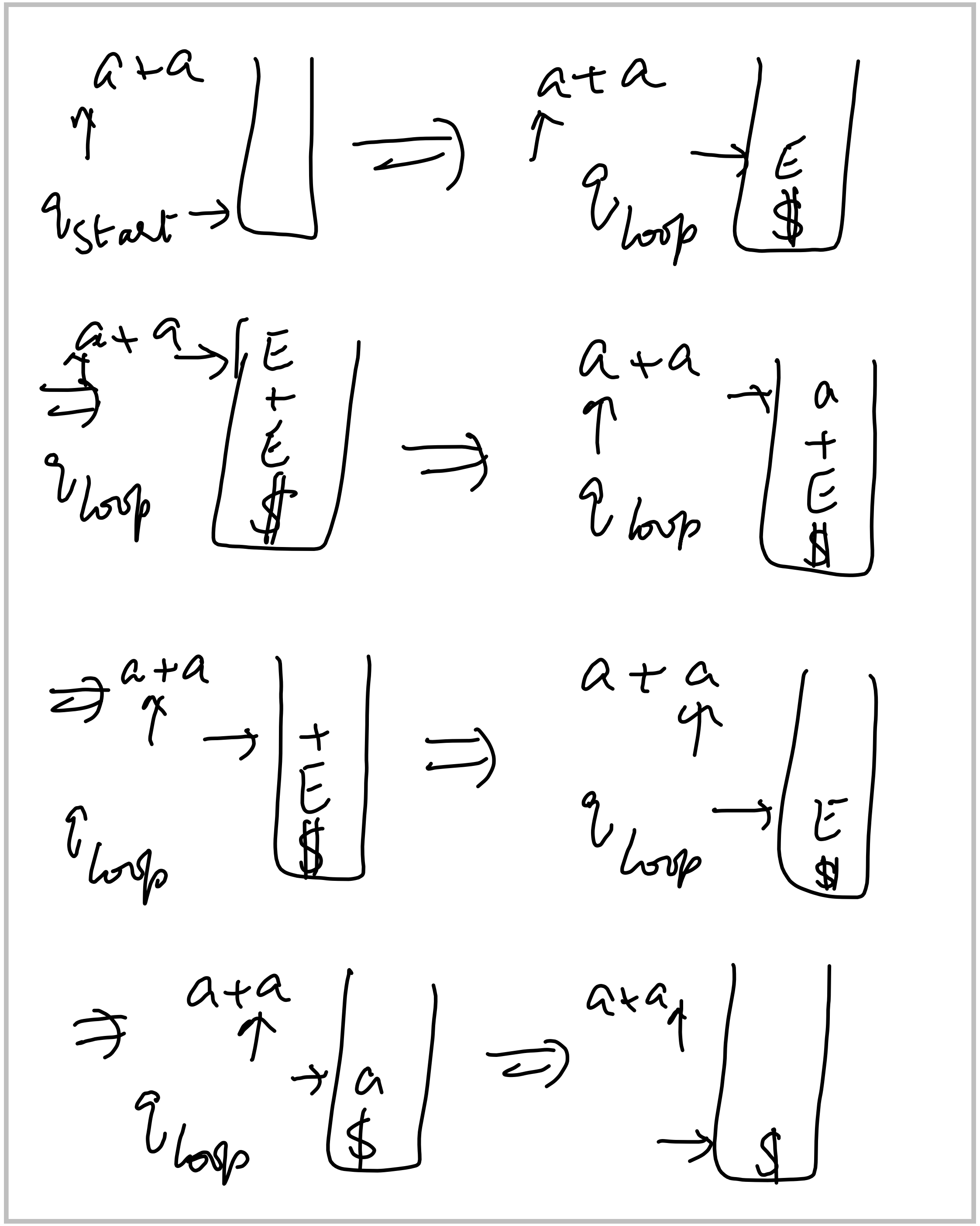
$$L(G) = \{ w \mid S \stackrel{*}{\Rightarrow} w \}$$

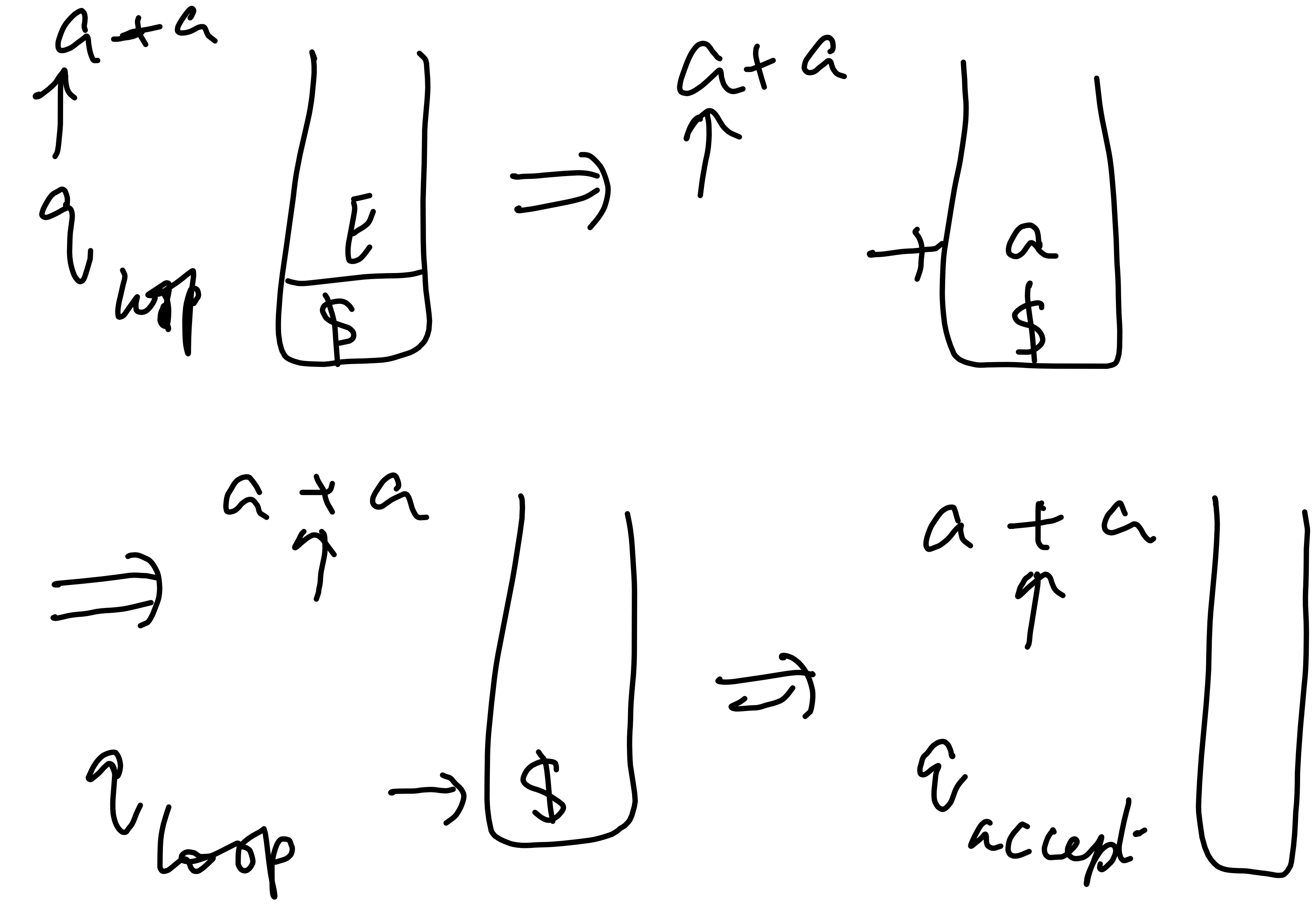
---

$$E \to E+E \mid E*E \mid (E) \mid a$$

$w$ : $a+a$

$$E \Rightarrow E+E \Rightarrow a+E \Rightarrow a+a$$


a+a
q_start
a+a
q_loop
E
$
a+a
E
+
E
$
q_loop
a+a
a
+
E
$
q_loop
a+a
+
E
$
q_loop
a+a
E
$
q_loop
a+a
a
$
q_loop
a+a
$


a+a
q loop
E
$
a+a
a
$
a+a
q loop
$
a+a
q accept

**Thm:** For every PDA $M$ there is a CFG $G$ s.t.

$$L(G) = L(M)$$

---

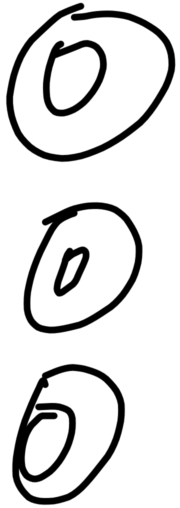
Given $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$

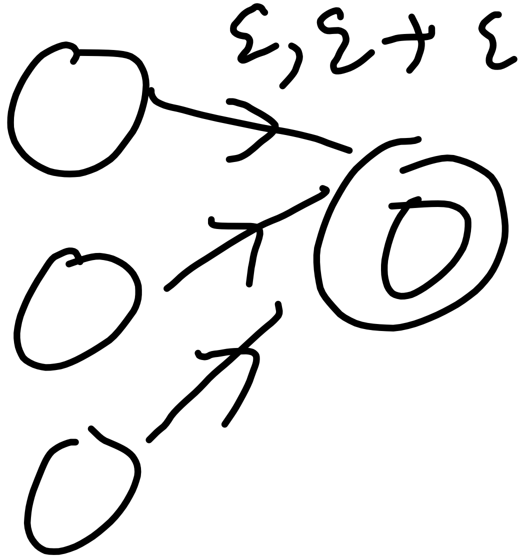
Construct $G = (V, \Sigma, R, S)$

s.t. $L(G) = L(M)$

Simplify machine M

(1) only one accept state

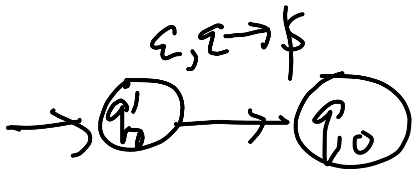




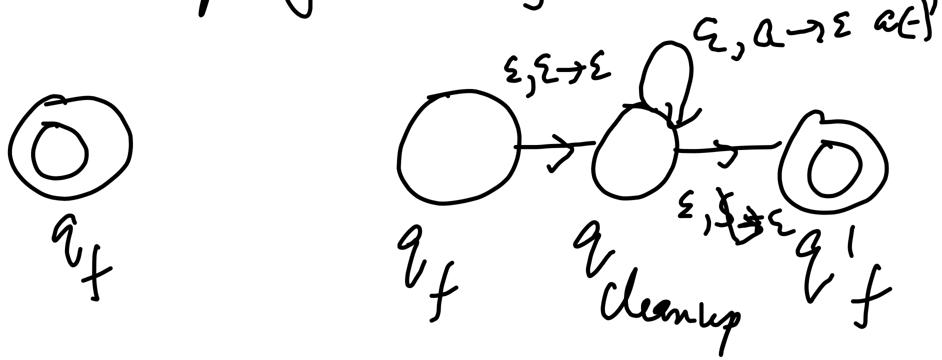
② M empties stack before accepting.

add \$ to $\Gamma$ ($\$ \notin \Gamma$)

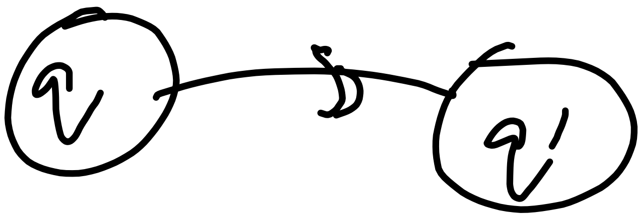
Create new start state that first pushes \$ onto stack

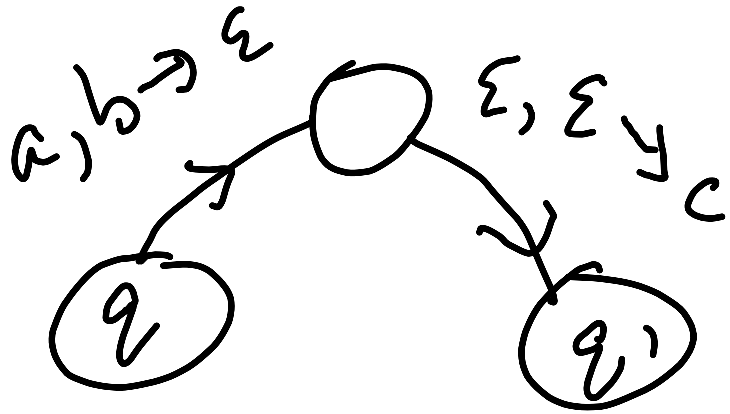



clean up from final state



(3) M on each transition either pushes onto stack & pop from stack but not both



$M = (Q, \Sigma, \Gamma, \delta, q_0, \{q_f\})$

also ③ implies

$\delta(q, a, b) \ni (q', c)$

$\Rightarrow$ either $b$ or $c$ is $\varepsilon$

$L(M) = \{ w \mid w$ takes $M$ from $q_0$ on empty stack to $q_f$ on empty stack$\}$

$S \overset{*}{\underset{G}{\Rightarrow}} w$ iff $w$ - - - -

$L_{pq} = \{ w \mid w$ takes $M$ on empty stack from $p$ to empty stack and $q \}$

$p, q \in Q$

$L(M) = L_{q_0 q_f}$

In grammar that we create a variable $A_{pq}$ to generate $L_{pq}$

Want to do

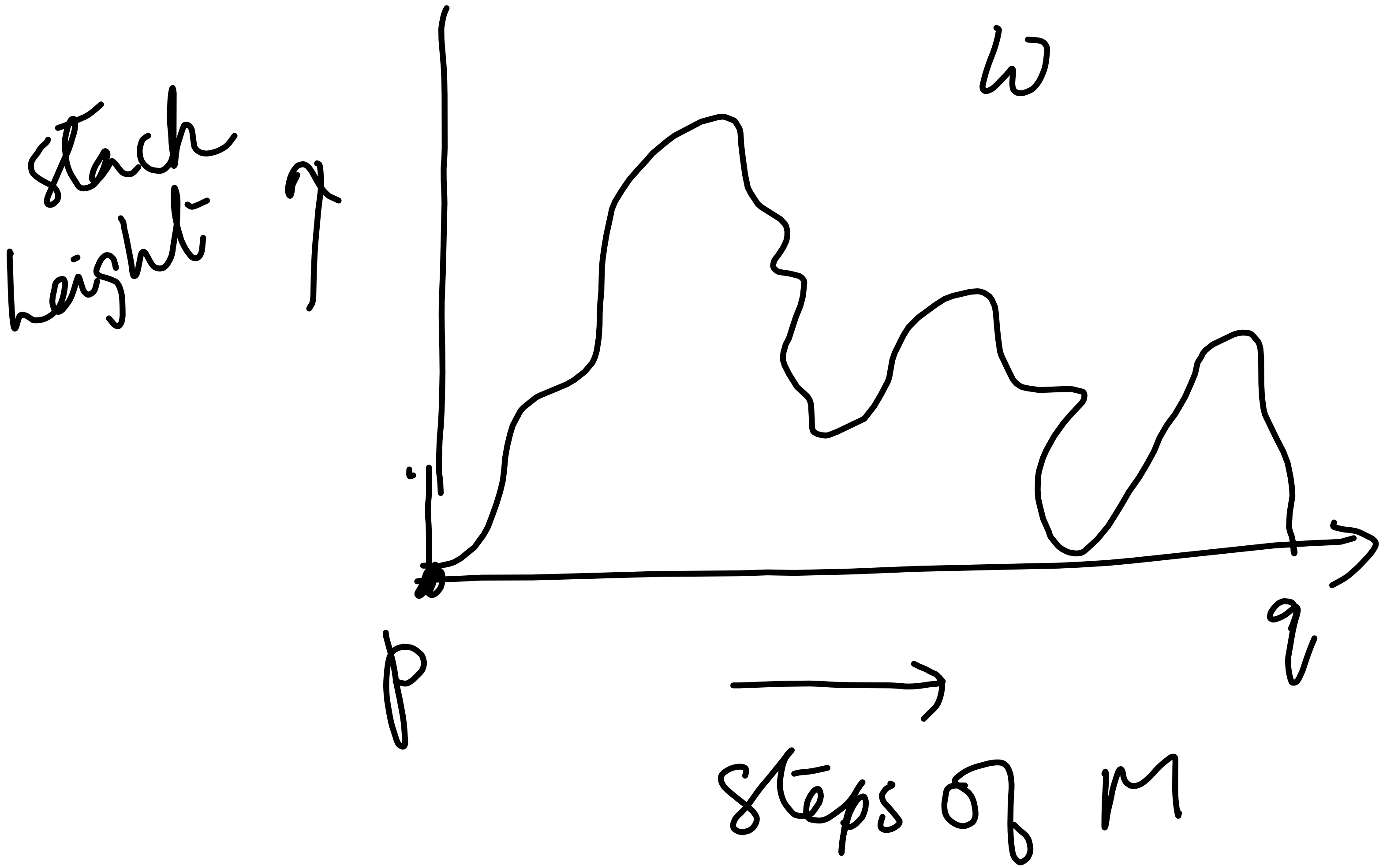
$A_{pq} \overset{*}{\Rightarrow} w$ iff $w \in L_{pq}$

$G = (V, \Sigma, R, S)$

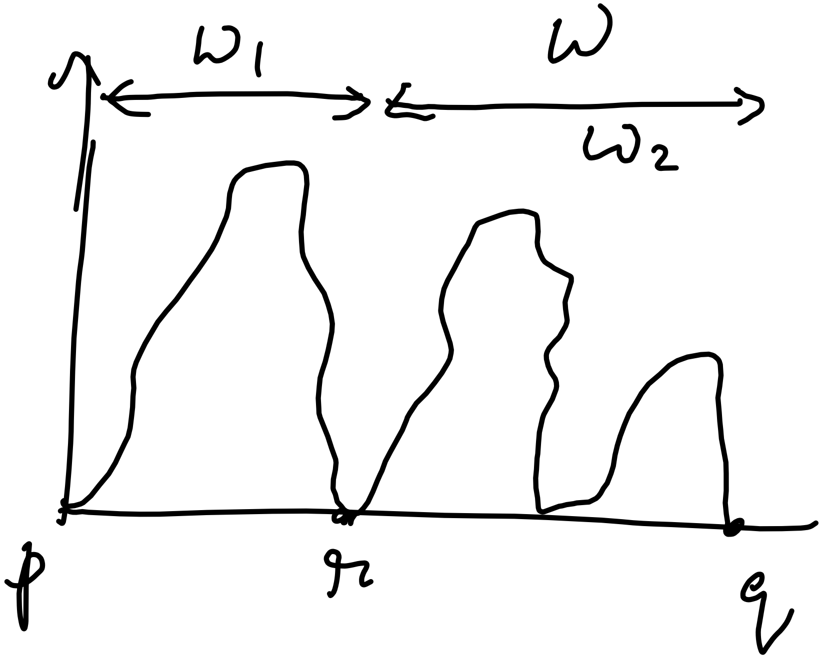
$V = \{A_{pq} \mid p, q \in Q\}$

$S = A_{q_0 q_f}$

Lpq



## Case 1

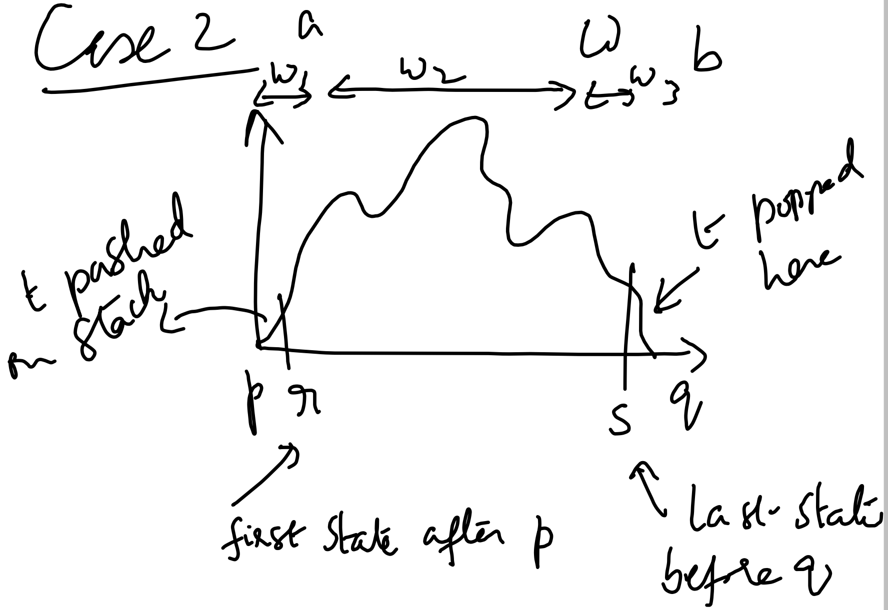



$$L_{pr} \cdot L_{rq} \subseteq L_{pq}$$

$$A_{pq} \to A_{pr} A_{rq}$$

add this for all $p, q, r$

## Case 2




$w = a w_2 b$

$w_2 \in L_{rs}$

① $(r, t) \in \delta(p, a, \varepsilon)$

② $(q, \varepsilon) \in \delta(s, b, t)$

$\forall\ a, b \in \Sigma_\varepsilon$ and $t \in \Gamma$

s.t.

$A_{pq} \to a\, A_{rs}\, b$

# Construction recap

$M = (Q, \Sigma, \Gamma, \delta, q_0, \{q_f\})$

Satisfies 1 2 3

Create $G = (V, \Sigma, R, S)$

$V = \{A_{pq} \mid p, q \in Q\}$

$S = A_{q_0 q_f}$

$R$: a) $A_{pq} \to A_{pr} A_{rq}$ $\forall p, q, r \in Q$

b) $A_{pq} \to a A_{rs} b$

$\forall a, b \in \Sigma_\varepsilon$ and $t \in \Gamma$

s.t. $(r, t) \in \delta(p, a, \varepsilon)$

and $(q, \varepsilon) \in \delta(s, b, t)$

c) $A_{pp} \to \varepsilon$ $\forall p \in Q$

You need to prove construction works

<u>To prove</u>

$$L(G) = L(M)$$

(a) $L(G) \subseteq L(M)$

(b) $L(M) \subseteq L(G)$

(a) $L(G) \subseteq L(M)$

$A_{q_0 q_f} \overset{*}{\Rightarrow} w$ implies $w \in L(M)$

---

$\forall\, p, q \in Q$

$A_{pq} \overset{*}{\Rightarrow} w$ implis $w \in L_{pq}$