CS 273 3/15/07

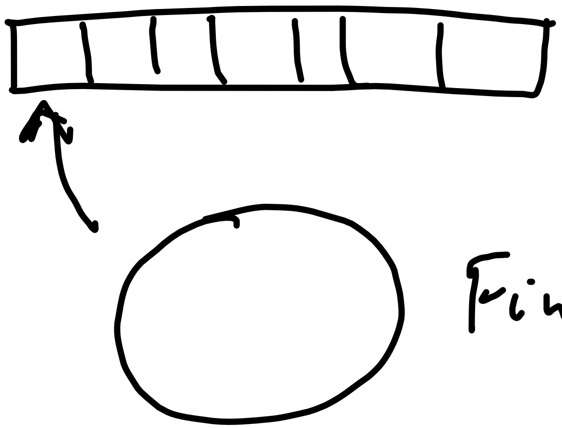
Turing Machines

- Alan Turing 1936
- Alonzo Church 1936
  λ-calculus

Formal definition of computation.

Finite State Machines

DFA's NFA's

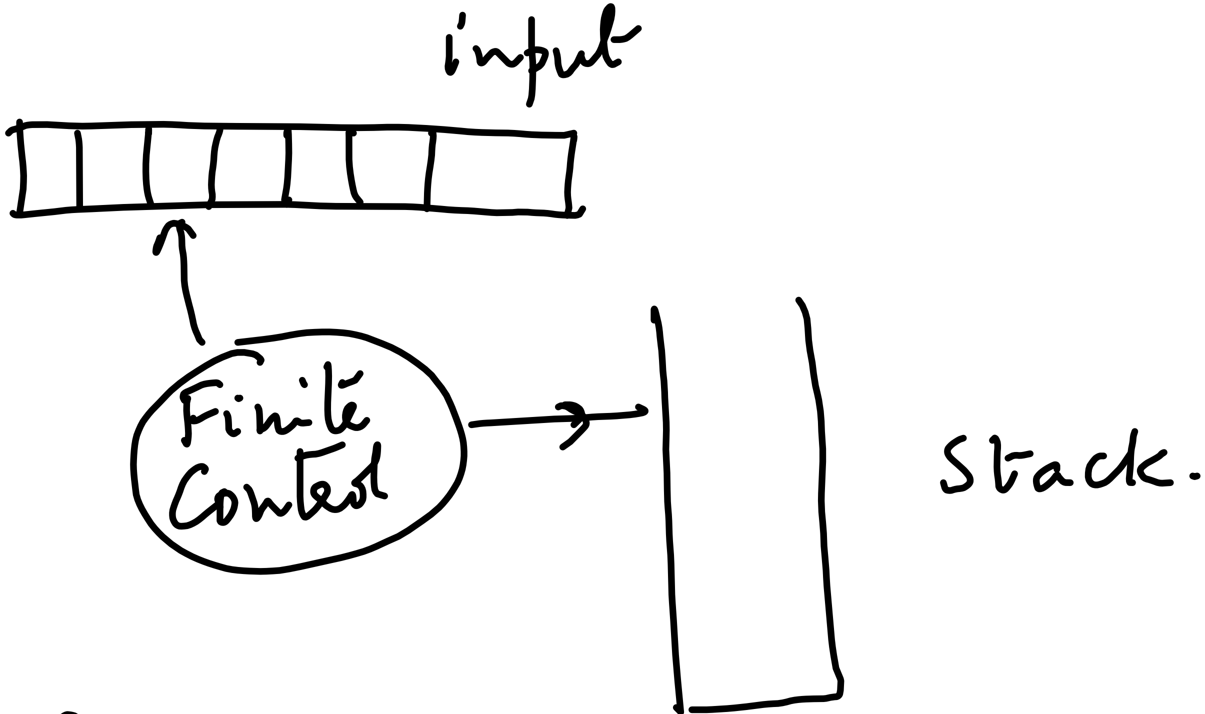


input

Finite Control
(States)

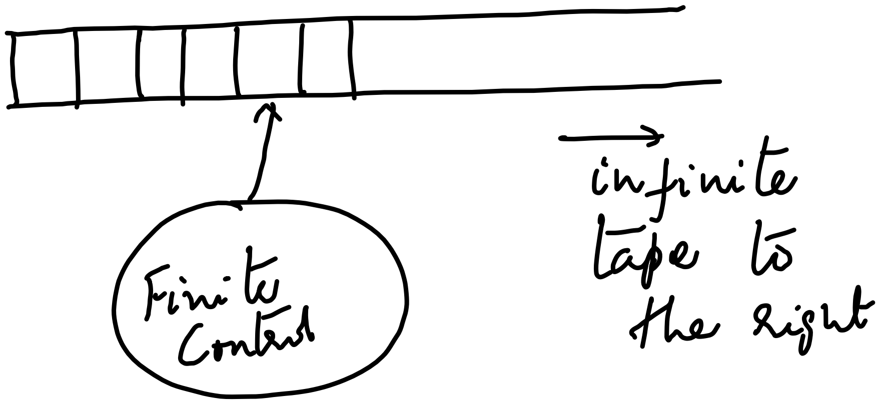
1. No extra memory
2. input is read left to right
   only once
3. all of input has to be read

PDA's



1. memory is stack
2. input has to be read completely from left to right

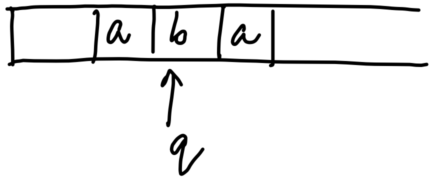
Turing Machines



1. infinite memory to the right
2. machine is allowed to move left and right on tape.
3. machine can read/write on tape
4. two special states q_{accept}, q_{reject}

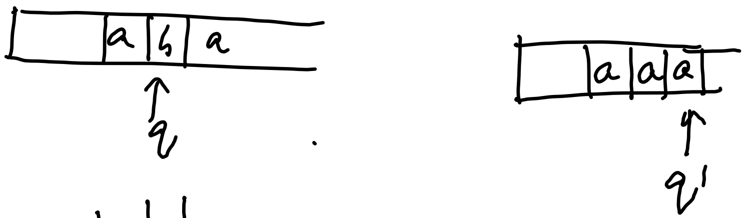
   – if machine enters these states it **halts**

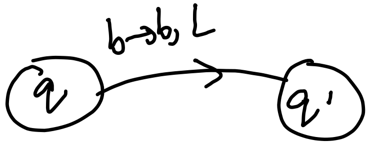
One move/step of a TM

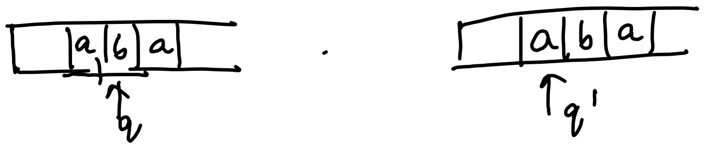


One step

- move to a new state
- move left (L) or right (R) on tape
- write a new symbol on current head location







Formally a TM is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$

- Q : ~~set~~ finite set of states

Σ : finite input alphabet

Γ : tape alphabet (finite)

$\Sigma \subseteq \Gamma$

Special blank symbol $\sqcup \in \Gamma$

$\sqcup \notin \Sigma$

q_0 : start state.

q_{accept} : single accept state

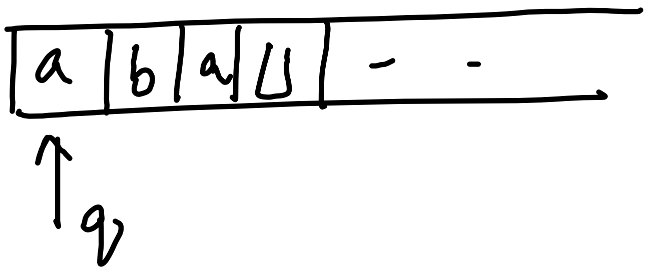
q_{reject} : single reject state

$\Big\}$ halting states

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$$

Two Notes

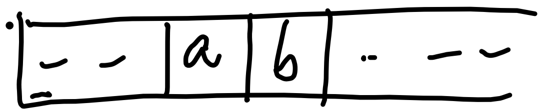
① What happens at the left end of tape?



Suppose $\delta(q, a) = (q', b, L)$

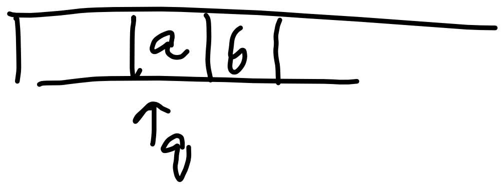
What happens?

(2) Machine can loop for ever



$$\delta(q, a) = (q', a, R)$$

$$\delta(q', b) = (q, b, L)$$



M has 3 types of behaviour

(1) enters q_{accept} and halts

(2) " q_{reject} and halts

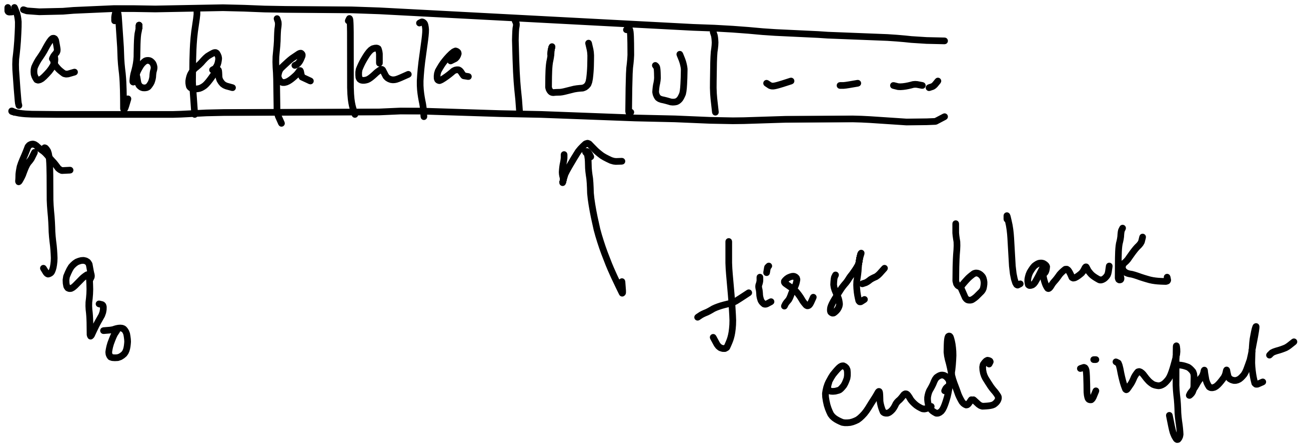
(3) loops forever

Example

$$L = \{ w \# w \mid w \in \{0,1\}^* \}$$

How does machine start?

How is input specified?



Example

$$L = \{ w\#w \mid w \in \{0,1\}^* \}$$

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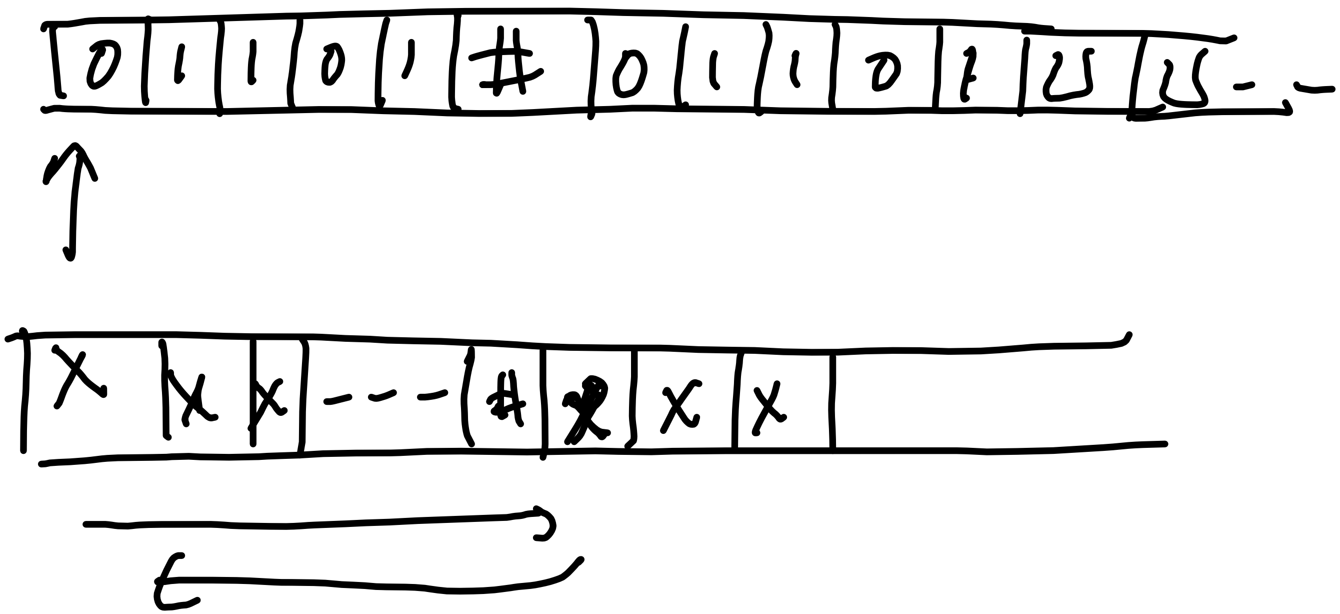
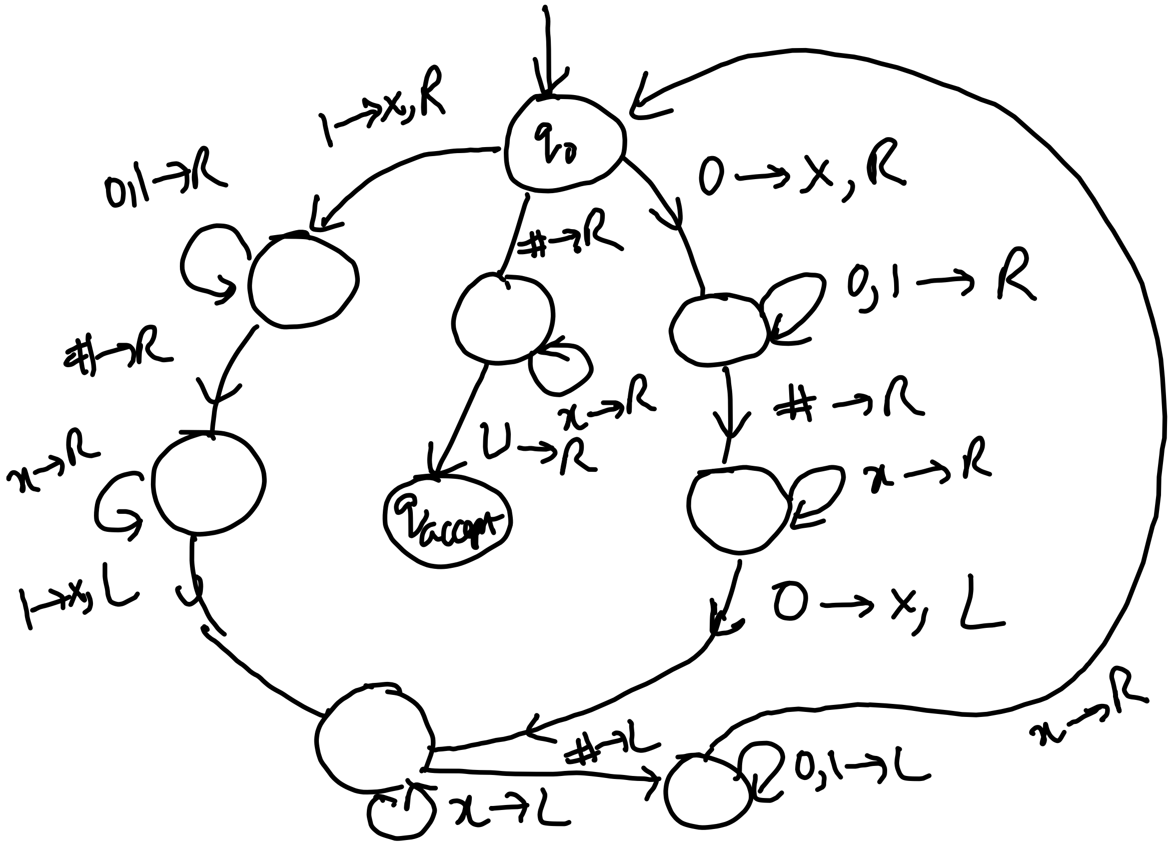


State diagram

$\Sigma = \{0, 1, \#\}$

$\Gamma = \{0, 1, \#, x, \sqcup\}$

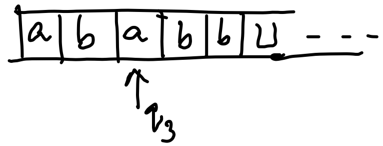


Formal defn of acceptance, language etc.

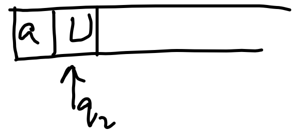
<u>Configuration of a machine</u>

- state
- contents of tape
- position of head.

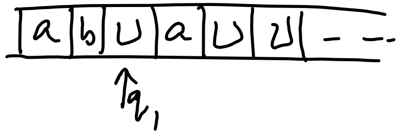
use $u\,q\,v$ $\qquad u, v \in \Gamma^*$



$ab\,q_3\,abb$



$a\,q_2$



$ab\,q_1 \sqcup a$

C_1 and C_2 are configurations

C_1 yields C_2 if M can move from C_1 to C_2 in one step.

$$C_1 \vdash_M C_2$$

$uaq\,bv$ yields $u\,q'acv$

iff $\delta(q,b) = (q', c, L)$

$uaq\,bv \vdash uacq'v$

iff $\delta(q,b) = (q', c, R)$

Accepting configuration

$u\, q_{accept}\, v$ for any $u, v \in \Gamma^*$

Rejecting config:

$u\, q_{reject}\, v$ for any $u, v \in \Gamma^*$

When does M accept w?

M accepts w if there exist configurations $C_1, C_2, \ldots, C_k$ s.t.

1. C_1 is (start) initial configuration $q_0 w$
2. C_i yields C_{i+1} $\quad i = 1, 2, \ldots, k-1$
3. C_k is an accepting configuration

Language of a machine M

$$L(M) = \{ w \mid M \text{ accepts } w \}$$

L is called Turing-recognizable if $\exists$ a TM M s.t.

$$L = L(M)$$

A TM M is called a <u>decider</u> if it halts on all inputs.

L is called **decidable**

if $L = L(M)$ for

a decider TM M.