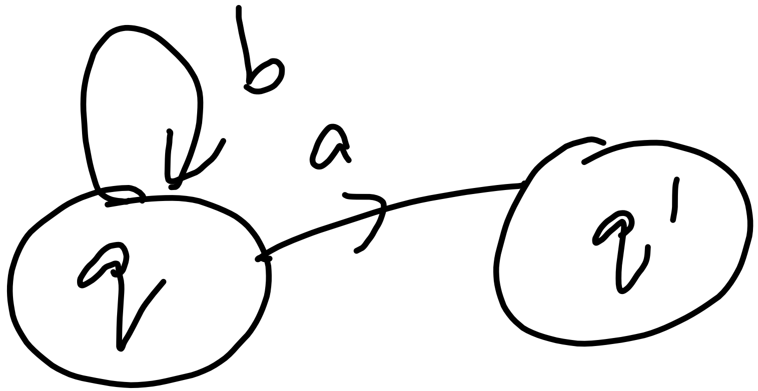
CS 273 1/30/07

Non Deterministic Finite Automata (NFA)

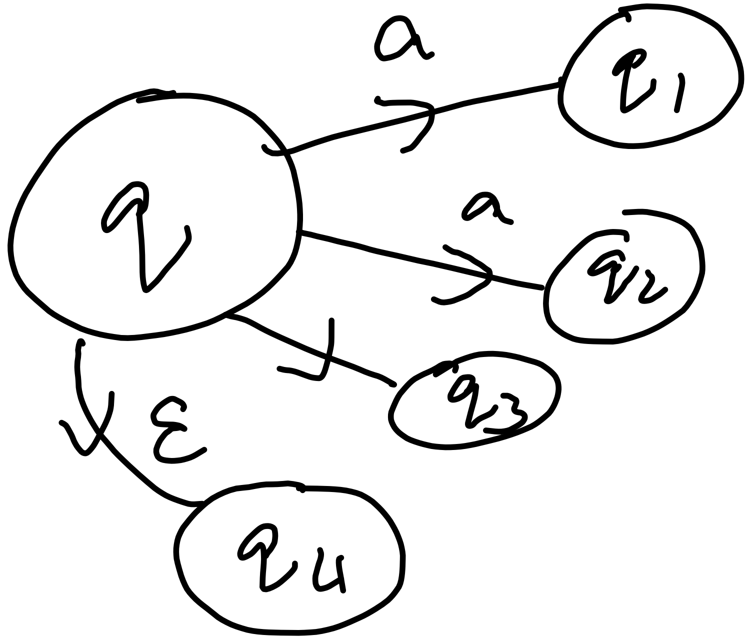
Reading: Section 1.2

DFA

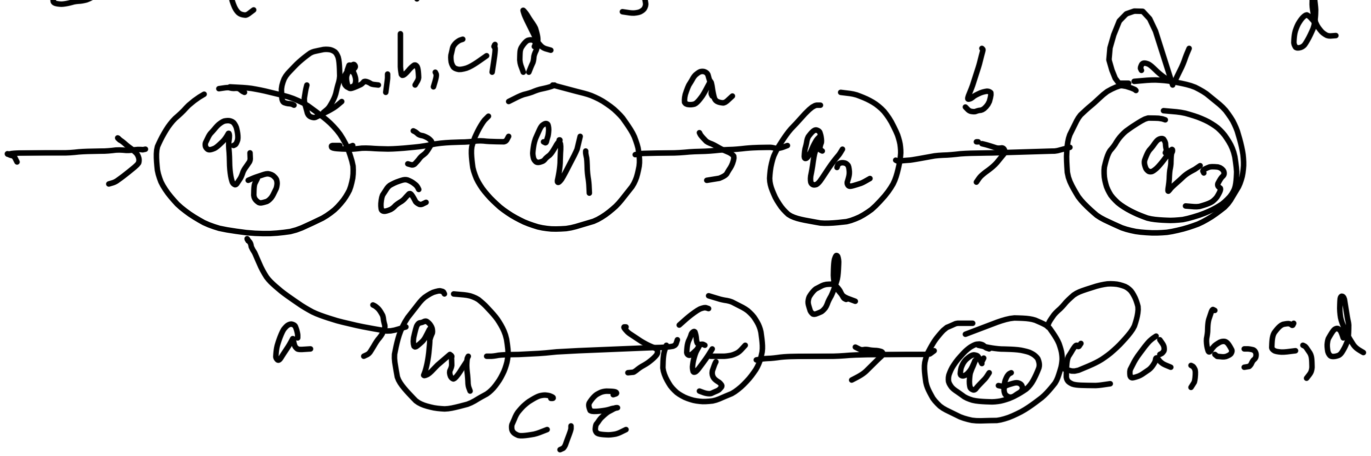


NFA

$\Sigma = \{a, b\}$



$\Sigma = \{a, b, c, d\}$



aaabc

$$q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{b} q_3 \xrightarrow{c} q_3$$

adb

$$q_0 \xrightarrow{a} q_4 \xrightarrow{\varepsilon} q_5 \xrightarrow{d} q_6 \xrightarrow{b} q_6$$

$$L = \{\ldots aab \ldots,\ \ldots acd \ldots,\ \ldots ad \ldots\}$$

1. Why NFA's

2. What can you do with them?

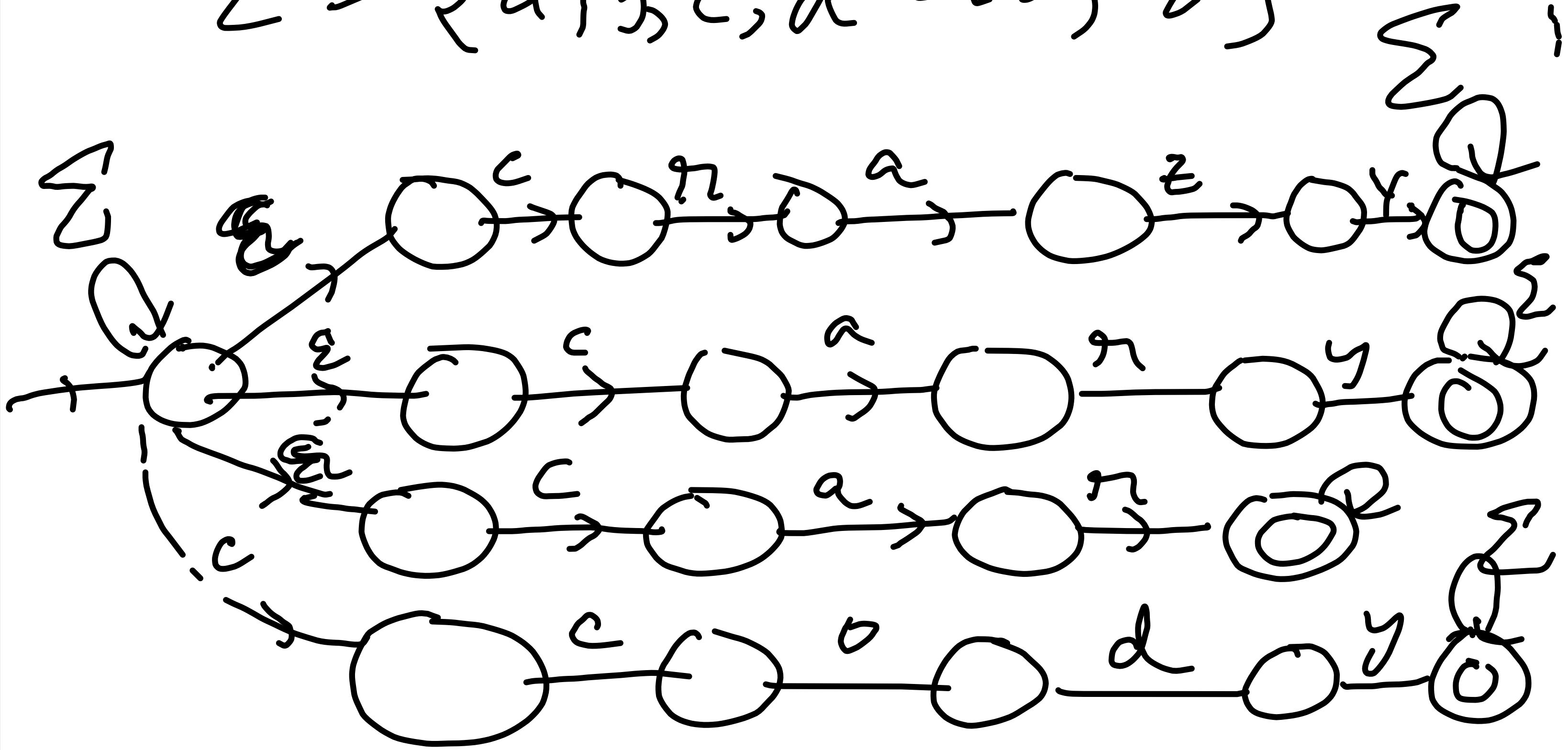
3. How can you run them on a computer?

$\forall$ NFA $\exists$ an equivalent DFA

1. NFAs are simpler and shorter to describe than DFAs
2. ~~a~~ easier to construct
3. certain closure properties are very easy to prove

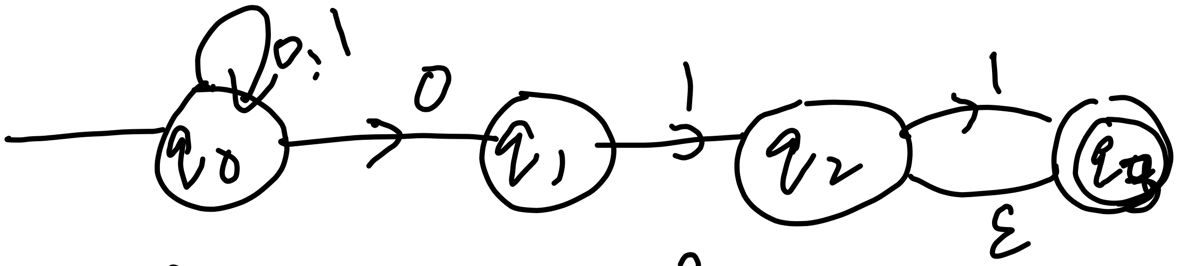
$L = \{ w \mid w$ contains "crazy" "cary", "cody", or "car" $\}$

$\Sigma = \{a, b, c, d, \ldots, z\}$



Formal defn of NFA.

5-tuple $N = (Q, \Sigma, \delta, q_0, F)$



$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0,1\}$

q_0

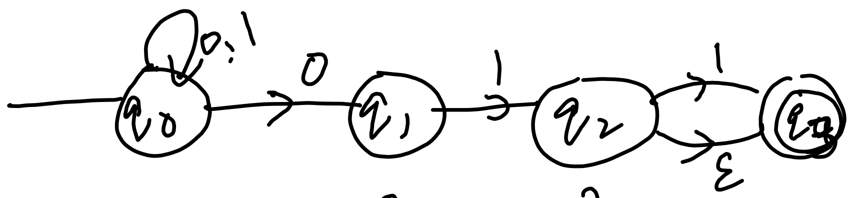
$F = \{q_3\}$

δ : ?

$\delta : Q \times \Sigma_\varepsilon \rightarrow \mathcal{P}(Q)$

Notation: $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$

$\mathcal{P}(Q)$



$$\delta(q_0, 0) = \{q_0, q_1\}$$
$$\delta(q_0, 1) = \{q_0\}$$
$$\delta(q_1, 0) = \phi$$
$$\delta(q_1, 1) = \{q_2\}$$
$$\delta(q_2, \varepsilon) = \{q_3\}$$

	ε	0	1
q_0	ϕ	$\{q_0, q_1\}$	$\{q_0\}$
q_1	ϕ	ϕ	$\{q_2\}$
q_2	$\{q_3\}$	ϕ	$\{q_3\}$
q_3	ϕ	ϕ	ϕ

Formal Defn of acceptance

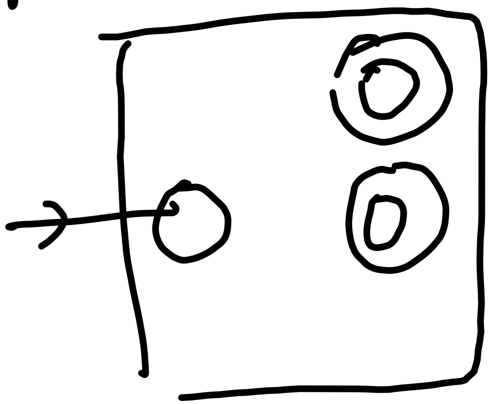
NFA $(Q, \Sigma, \delta, q_0, F)$ accepts a string $w \in \Sigma^*$ iff there is a sequence of symbols $y_1 y_2 \cdots y_m$ $\quad y_i \in \Sigma_\varepsilon$ and there is a ~~set~~ sequence of states $r_0 r_1 \cdots r_m$ s.t.

1. $w = y_1 y_2 \cdots y_m$
2. $r_0 = q_0$
3. $r_m \in F$.
4. $r_{i+1} \in \delta(r_i, y_{i+1}) \qquad i = 0, 1, 2, \ldots, m-1$

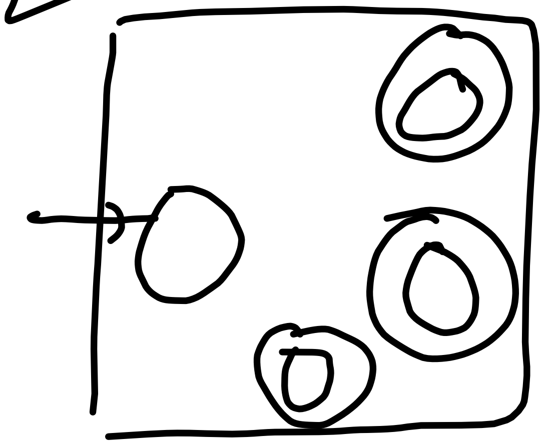
Closure Properties of NFAs

1. L_1 is a language of NFA N_1
and L_2 is language of N_2
$\Rightarrow$ there is an NFA N for $L_1 \cup L_2$

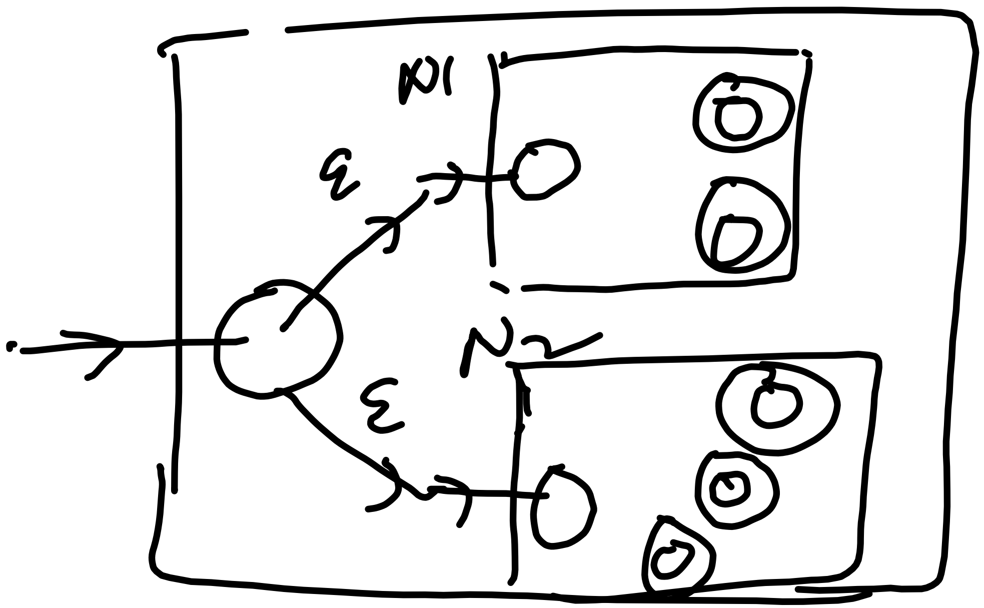
N1



N2



N

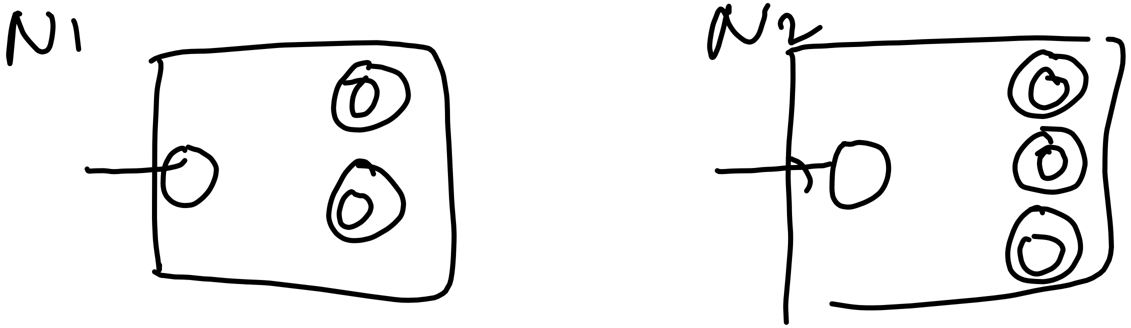


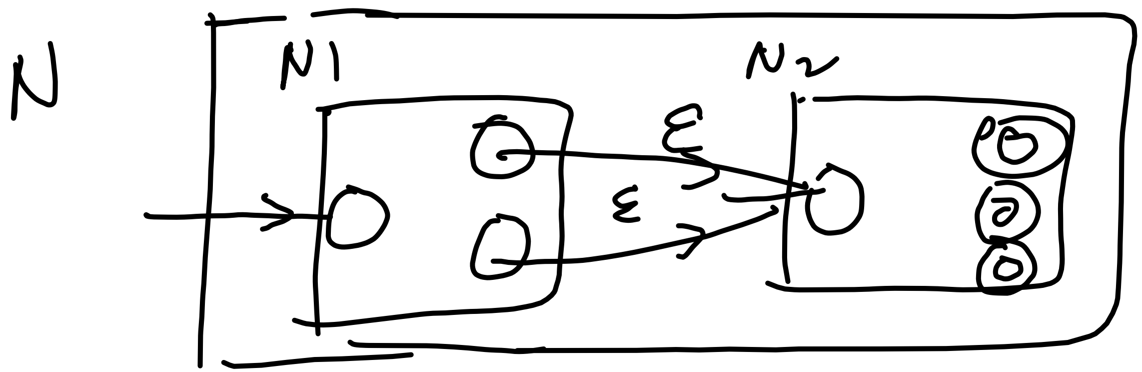
Concatenation

$L_1 \circ L_2 = \{xy \mid x \in L_1, y \in L_2\}$

$L_1 = L(N_1)$ and $L_2 = L(N_2)$

$\Rightarrow \exists N$ s.t. $L(N) = L_1 \circ L_2$





closure under star

Given language L

$$L^* = \{ w_1 w_2 \cdots w_k \mid w_i \in L, i = 1, 2, \ldots, k \}$$

Kleene Star

$$L = \{a\}$$

$$L^* = \{ \varepsilon, a, aa, aaa, \ldots \}$$

$$L = \{ a, abc \}$$

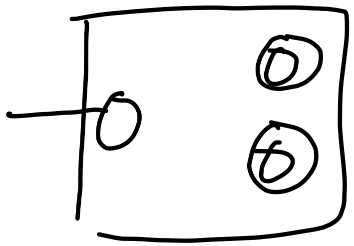
$$L^* = \{ \varepsilon, a, aa, \ldots, abc, abcabc, aabc, \ldots \}$$

L has an NFA

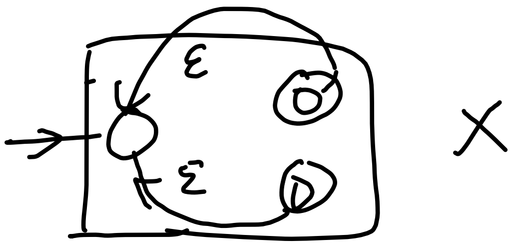
$\Rightarrow L^*$ has an NFA

say $L = L(N_1)$

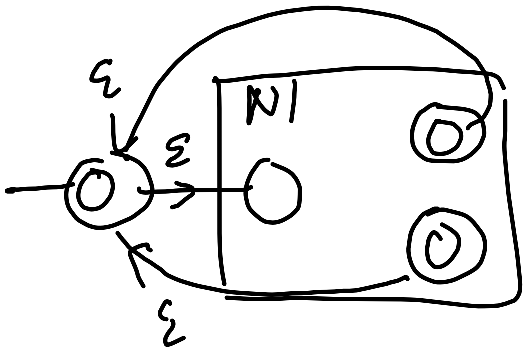
N_1



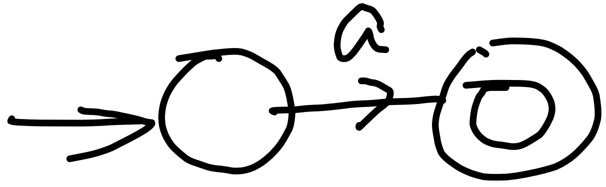
N



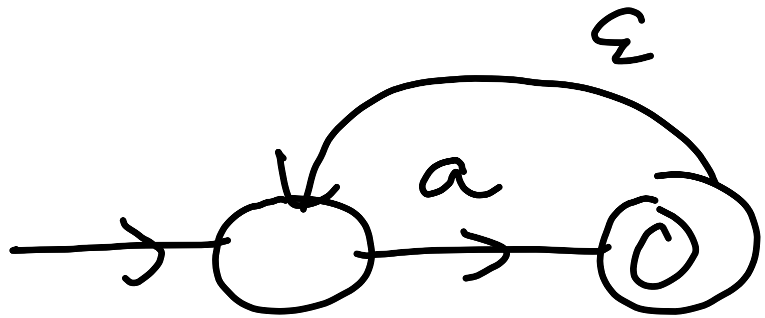
N



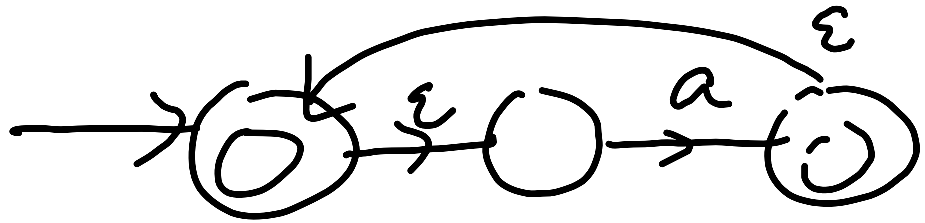
$L = \{a\}$



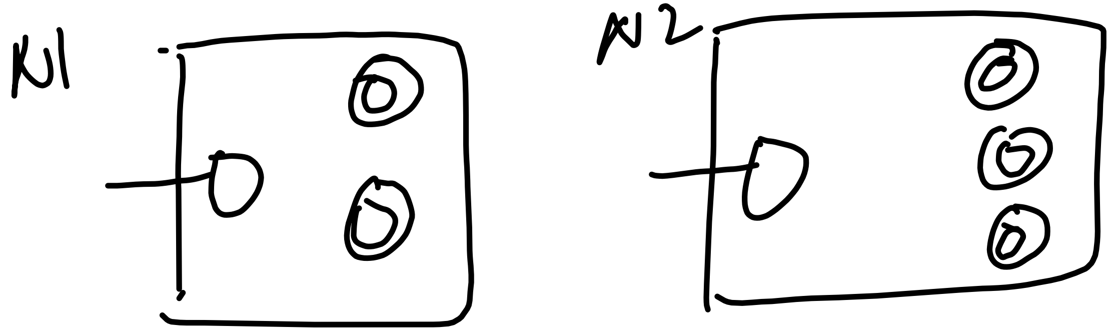
$L^* = \{\varepsilon, a, aa, \ldots\}$

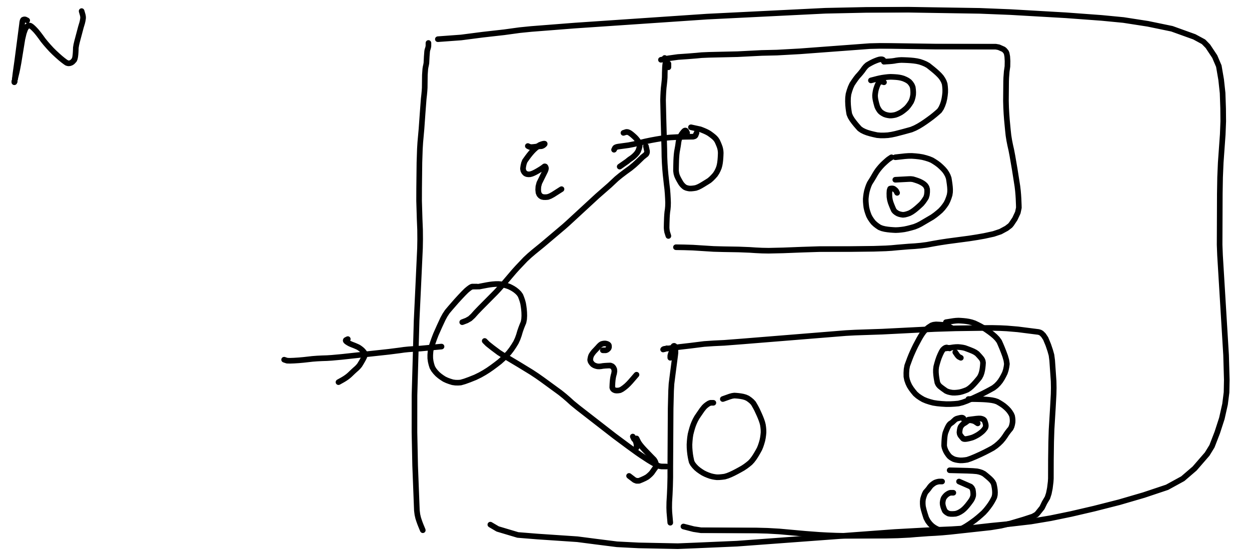


Does not accept ε



Notation for N that accepts
$L(N_1) \cup L(N_2)$





$N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$

$N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$

$N = (Q, \Sigma, \delta, q_s, F)$

$Q = Q_1 \cup Q_2 \cup \{q_s\}$

$F = F_1 \cup F_2$

δ: ?

$\delta(q_s, \varepsilon) = \{q_1, q_2\}$

$\delta(q_s, a) = \phi \qquad a \in \Sigma$

$\delta(q, a) = \delta_1(q, a) \qquad q \in Q_1, \; a \in \Sigma_\varepsilon$

$\delta(q, a) = \delta_2(q, a) \qquad q \in Q_2, \; a \in \Sigma_\varepsilon$