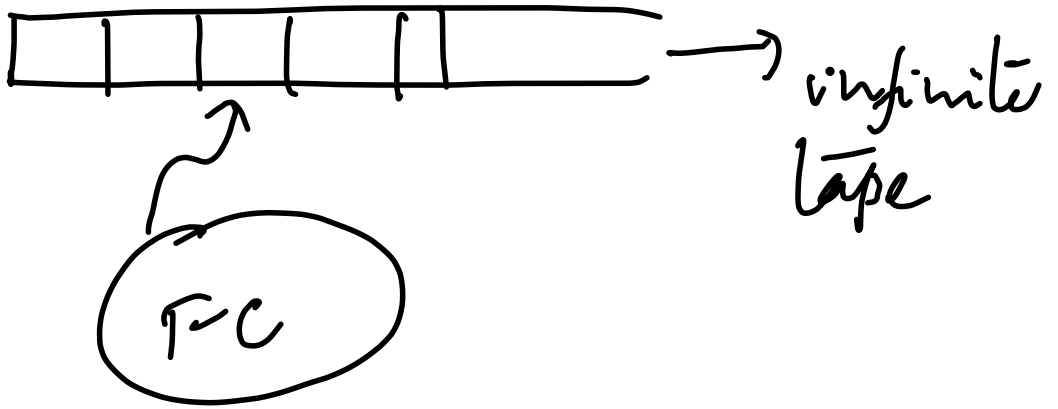
CS 273 3/27/07

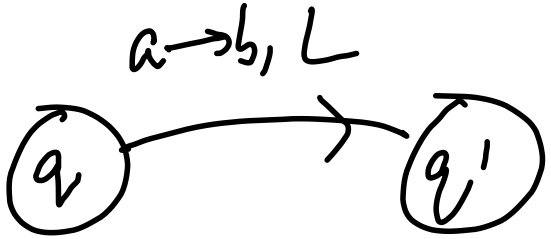
- Recap of TMs
- Multi tape TMs
- Encoding Problems
- Decidability

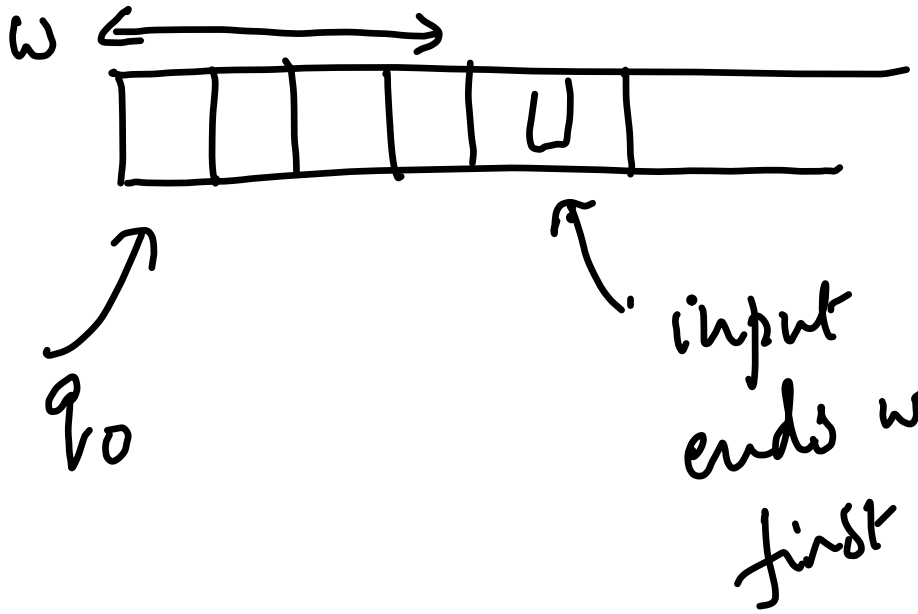
# TM




$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$$

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$$






Starting Configuration

$$L(M) = \{ w \mid M \text{ halts in } q_{accept} \text{ from } q_0 w \}$$

Configuration: $\alpha q \beta$

M on w

- can accept (halt)
- can halt and reject
- loop forever

A language L is called Turing recognizable if there is a TM M s.t for $w \in L$, TM accepts

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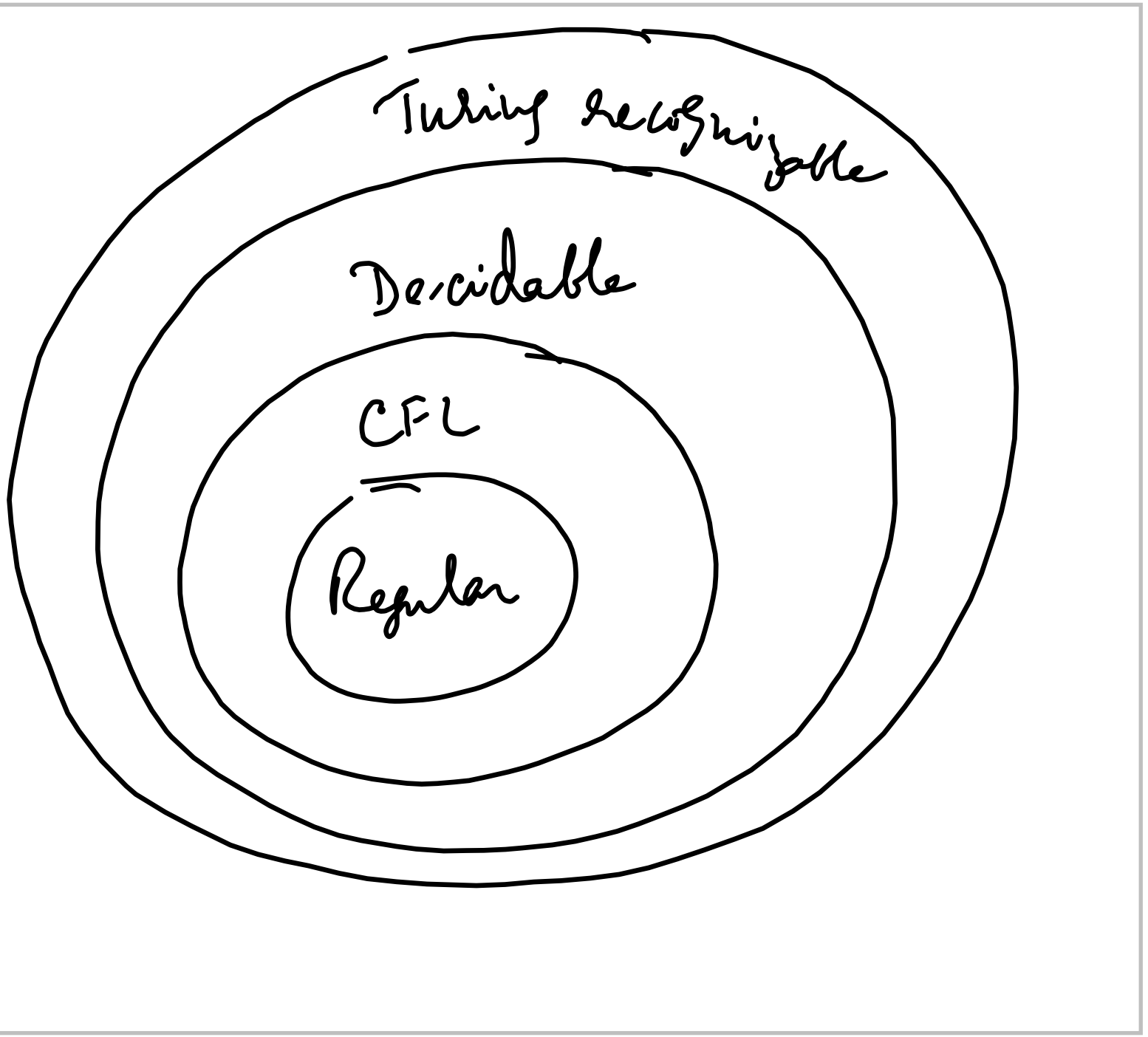
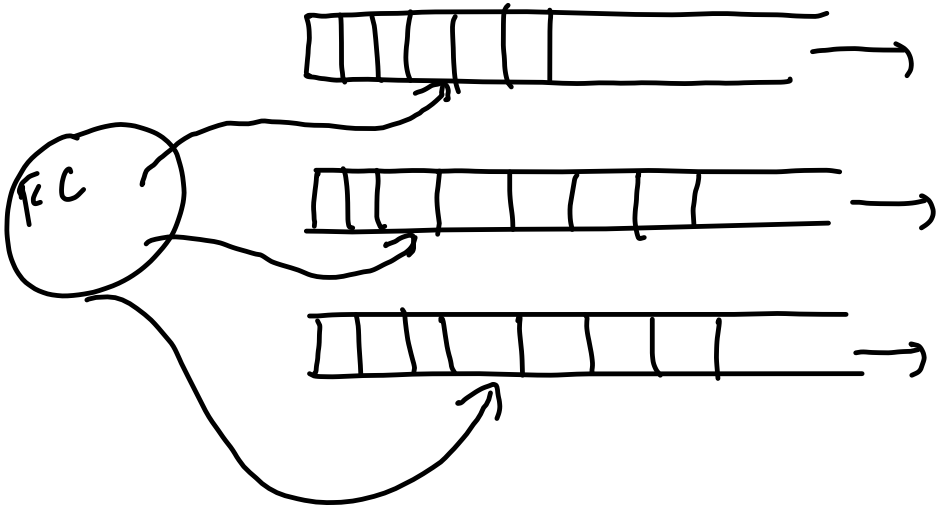
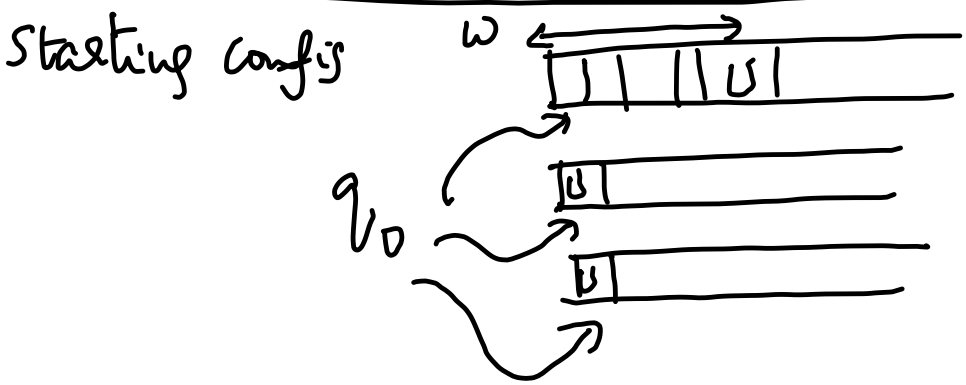
A language L is called <u>decidable</u> if there is a TM M that always halts on any input and accepts L.


Turing recognizable
Decidable
CFL
Regular

# Multi tape TMs



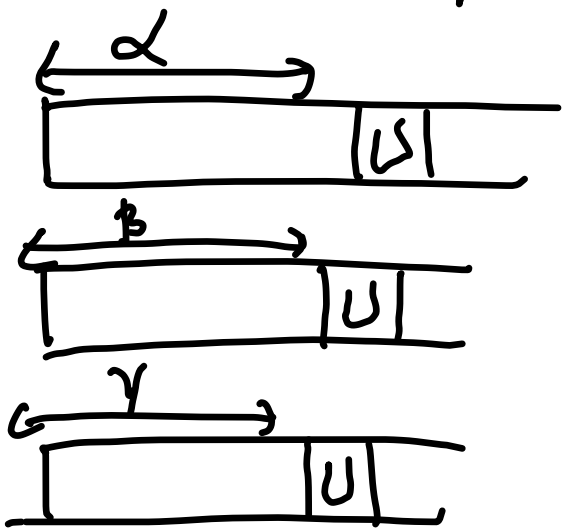

$$\delta : Q \times \Gamma^k \longrightarrow Q \times \Gamma^k \times \{L, R\}^k$$

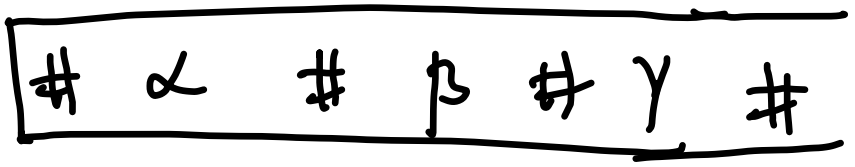


**Thm:** If $L$ is recognized by a $k$-tape TM $M$ then $L$ is recognized by a 1-tape (regular) TM $M'$.

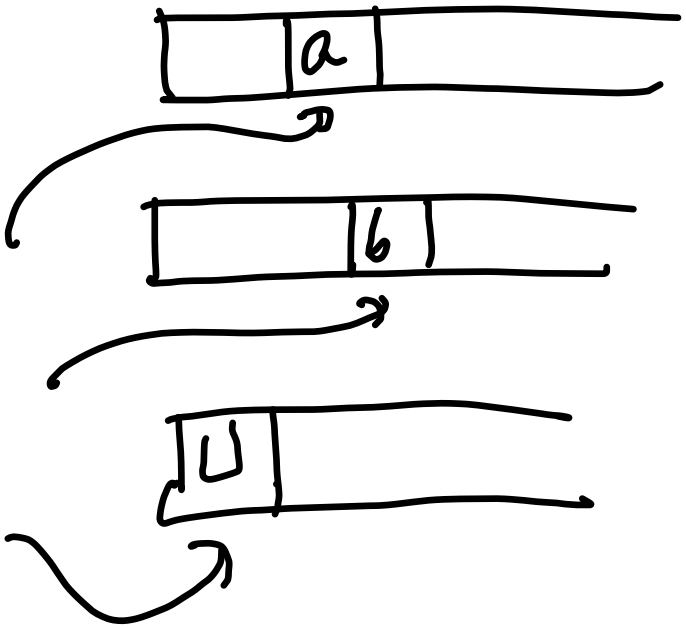
- Show how $k$ tapes can be stored on 1 tape
- Show how to keep track of head positions on $k$ tapes
- Show how to simulate each move of $M$ in $M'$ (using potentially many moves)

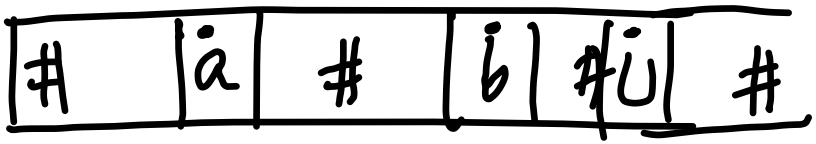
① How to keep track of k tapes on 1 tape



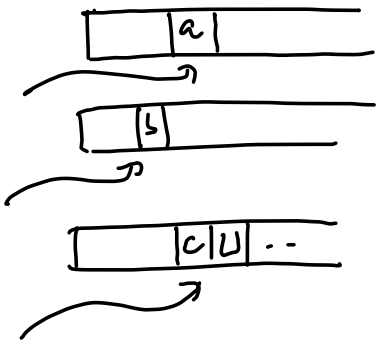



## (2) How to keep track of head positions



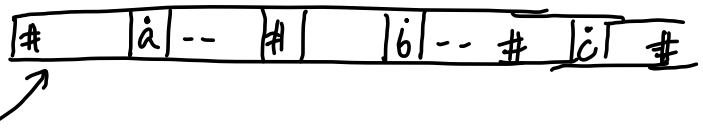
(3) How to simulate one move of $M$ in $M'$




$\delta(q, a, b, c)$
$= (q', a', b', c', L, R, L)$

$M$

$M'$



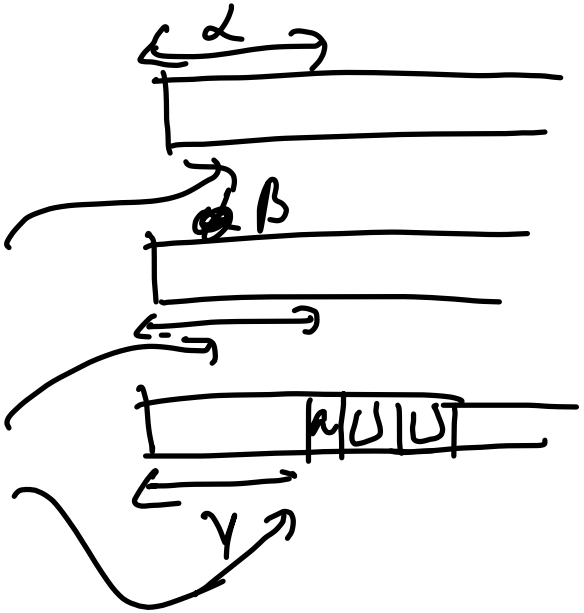

- $M'$ has to remember state of $M$ (in its state).
- Scans tape from left to right to find tape symbols that $M$ sees. (remember tape symbols in state).
- Come back to beginning of tape so now $M'$ knows what $M$ should do.

- Scans tape again
  - if it sees a symbol with a • replace the symbol as M would do.
  - move head L or R as M would do – put a • on left & right
- Trouble?? (next slide)

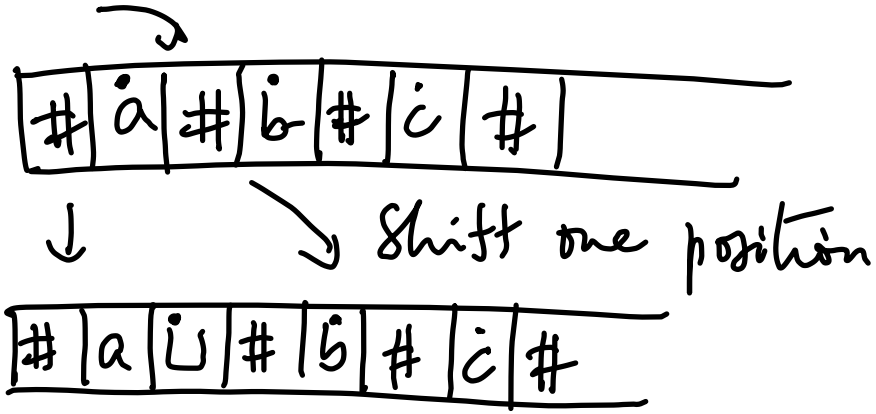
- Come back to beginning of tape

④ Accept / Reject if M accepts/rejects

# Trouble







How many steps does $M'$ take for 1 step of $M$. ?

If $M$ takes $n$ steps then $M'$ takes $O(kn^3)$ steps

# Church-Turing thesis

TM — algorithm.

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Hilbert's 10th problem

$$3x^3y + y^2z + z^3 = 0$$

does this have integer solutions?

① You can encode TMs as strings!

## Encoding problems

Any problem you have has to be cast as a language recognition problem.

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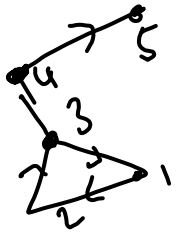
Given a directed graph $G$ and two nodes $s, t$.
Does $s$ have a path to $t$?

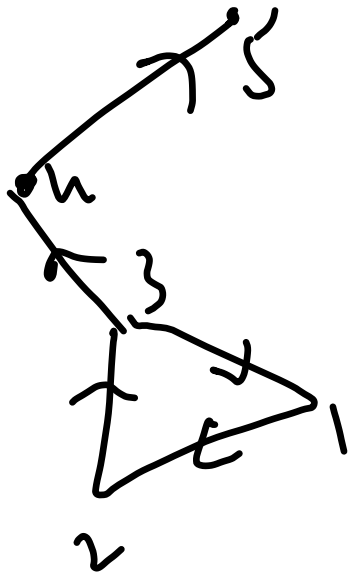



$G$ $\quad s = 2, \; t = 5$

$$L = \{ \langle G, s, t \rangle \mid G \text{ is an encoding of a graph and } s \text{ has a path to } t \text{ in } G \}$$

Claim $L$ is <u>decidable</u>

Encoding $\langle G, s, t \rangle$




$1, 2, 3, 4, 5$; $(4,2), (2,3), (3,1), (3,4), (4,5)$; $2$; $5$

1. Mark $s$

2. Repeat

2.a for each edge $(u,v)$
if $u$ is marked and $v$ is not marked

mark $v$

3. Until some new node is marked

4. Say Yes if $t$ is marked
otherwise no.

Any regular language $L$ is decidable.

Take a DFA $M$ for $L$

Create TM $M'$ that works like $M$

$M = (Q, \Sigma, \delta, q_0, F)$

$M' = (Q \cup \{q_{accept}, q_{reject}\}, \Sigma, \Gamma, \delta', q_0, \dots)$

$\delta'(q, a) = (\delta(q,a), a, R)$ $a \in \Sigma$

$\delta(q, \sqcup) = q_{accept}$ $q \in F$

$= q_{reject}$ $q \notin F$

On TM