HW 1 due tomorrow 3^{30}

web survey closes "tonight"

HBS should start next week

Are regular lgs closed under union?

If L_1 is regular &

L_2 is regular,

is $L_1 \cup L_2$ always regular?

DFAs

NFAs

regular expression

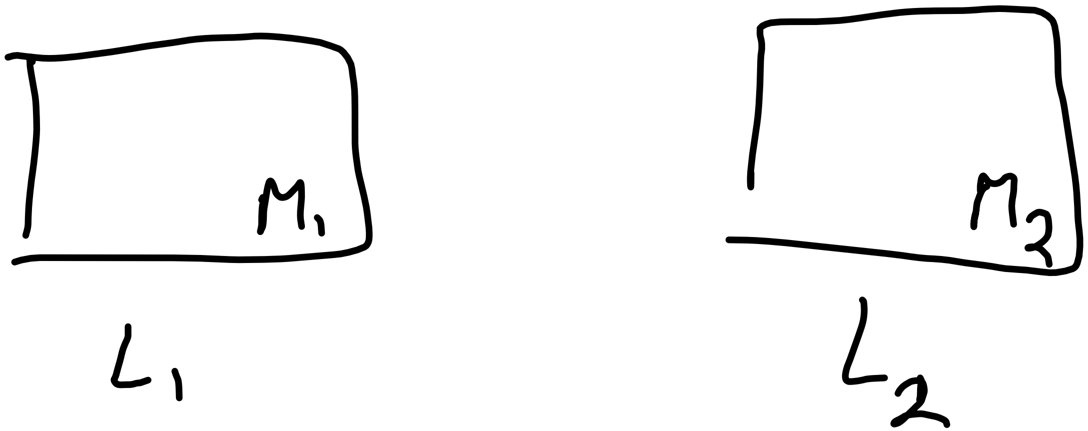
Math answer

Closure properties

Proofs easier to construct

Application answer

machines composable



easily adaptable

Latin-1 $\Rightarrow$ unicode

"closure under homomorphism"

efficiency

Regular lg closure properties

- union
- intersection } go together
- complement }

$$A \cap B = \overline{(\bar{A} \cup \bar{B})}$$

regular ops

- concatenation
- star
- homomorphism
- reverse

$L_1 \circ L_2$

$$= \{ xy : x \in L_1 \ \& \ y \in L_2 \}$$

$$L^* = \{ w_1 \dots w_n \mid n \geq 0, \ w_i \in L \ \forall i \}$$

"star"

$$w^* = \{ \varepsilon, w, ww, www, \dots \}$$

$$= \{ w_1 w_2 \dots w_n \mid w_i = w \ \forall i \ \& \ n \geq 0 \}$$

Thm: Regular lgs are closed under union

Proof: Let L_1 & L_2 be regular lgs.

So there are DFA's M_1 & M_2 recognizing L_1 & L_2

$$M_1 = (Q, \Sigma, \delta, q_0, F)$$

$$M_2 = (Q', \Sigma, \delta', q_0', F')$$

New machine N has

set of states $Q \times Q'$

start state (q_0, q_0')

final states $\{(p,q) \mid p \in Q, q \in Q', \text{ and } (p \in F \text{ or } q \in F')\} = \hat{F}$

Transition function $\hat{\delta}$

$$\hat{\delta}((p,q), a) =$$

$$(\delta(p,a), \quad \leftarrow \text{state of } M$$

$$\delta'(q,a)) \quad \leftarrow \text{state of } M'$$

$$\delta(p, a)$$

$$Q \nearrow p, \quad \Sigma \uparrow a$$

So new machine $\hat{M}$

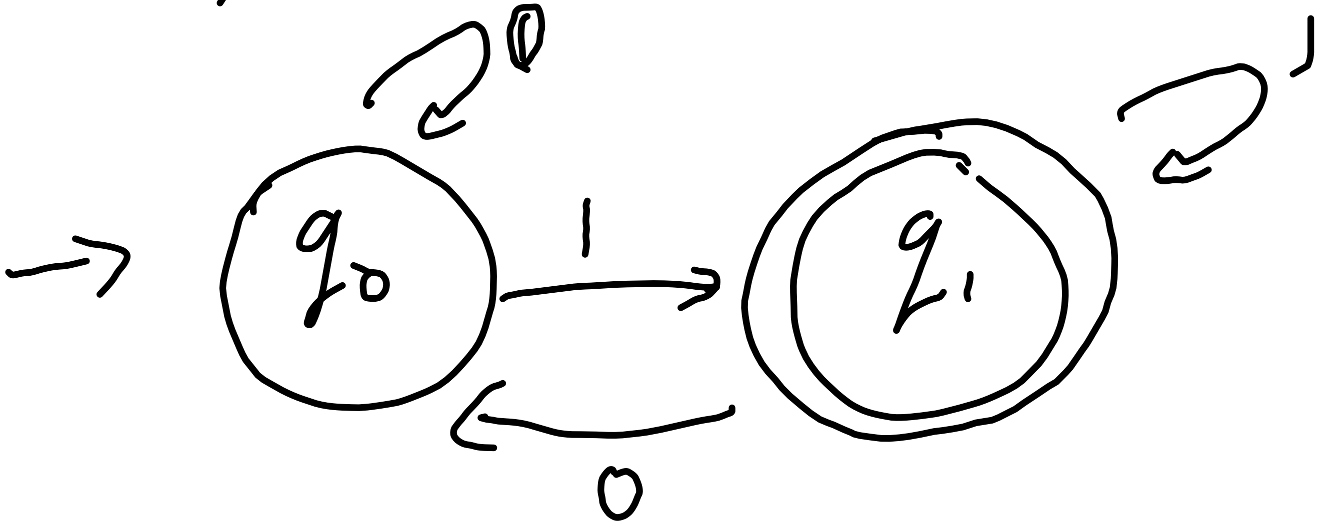
$$= \left(Q \times Q', \Sigma, \hat{\delta}, (q_0, q_0'), \hat{F} \right)$$

defined above

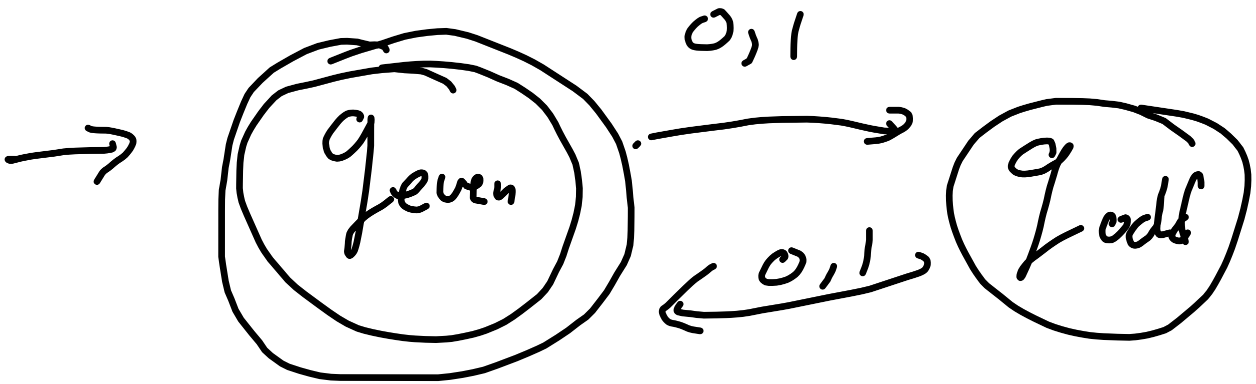
recognize $L_1 \cup L_2$

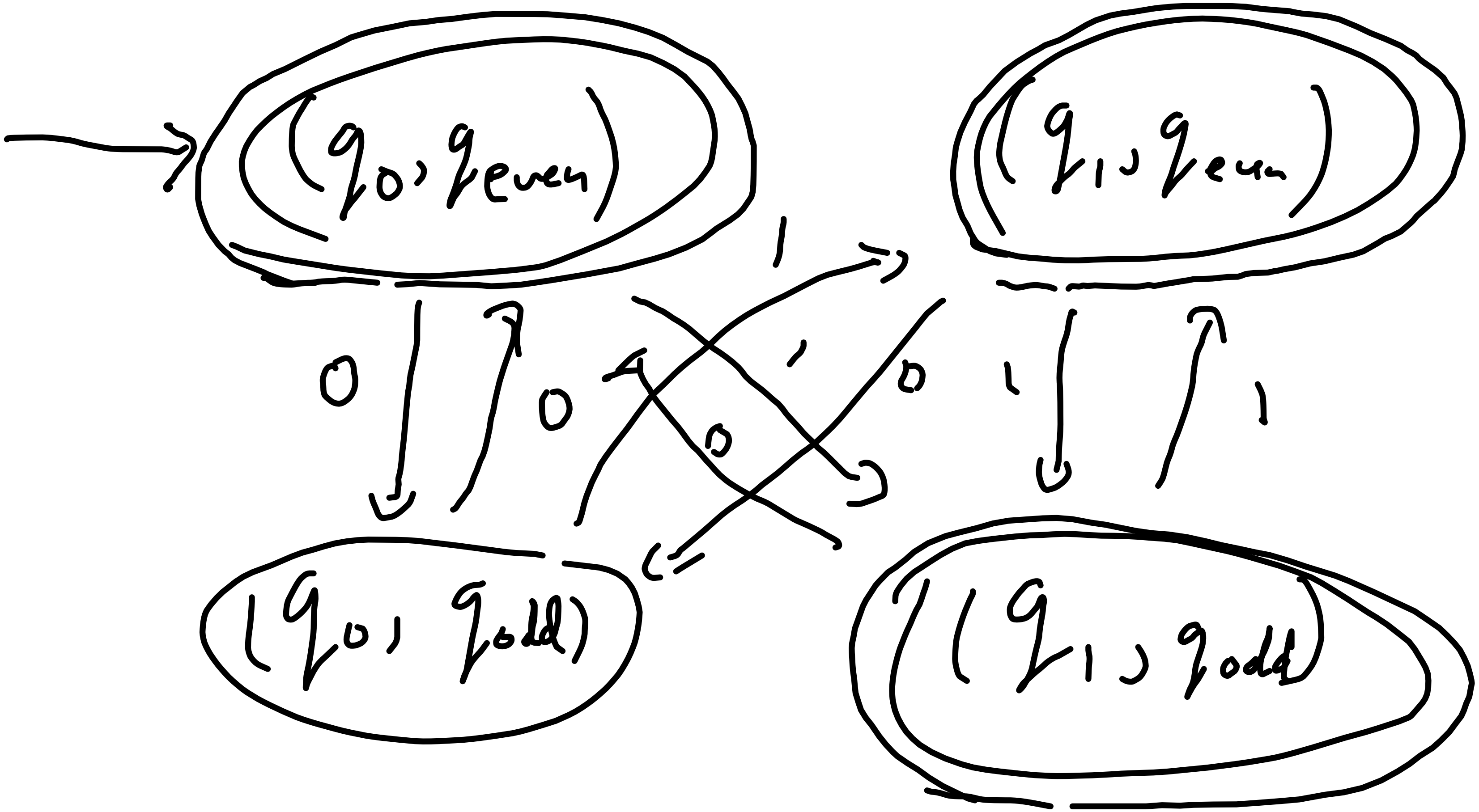
$\Sigma = \{0,1\}$

M_1



M_2




(q0, qeven)
(q1, qeven)
(q0, qodd)
(q1, qodd)
0
0
1
1
0
0
1
1

(e, a)

look at all input chars

$$e \xrightarrow{1} g$$
$$a \xrightarrow{1} b$$

marked in table

(e, a) are distinct because
(g, b) are distinct

(a, h)

$a \xrightarrow{1} b$

$h \xrightarrow{1} c$

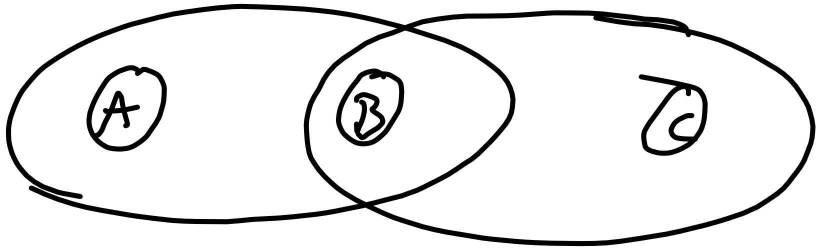
marked in table

so since (b, c) is known distinct

(a, h) are distinct

Questions?

- Does this really produce minimal DFA for this language?
- Is "not distinct" an equivalence relation?



- How long does it take?

looks like $O(n^4)$ $n = \#$ states

can be $O(n \lg n)$