# Post's Correspondence Problem

# &

# Tilings

# PCP

set of dominos $\left[\dfrac{\alpha}{\beta}\right]$ ← 2 strings

decision problem
given set of dominos

$$S = \left\{ \left[\frac{b}{ca}\right], \left[\frac{a}{ab}\right], \left[\frac{ca}{a}\right], \left[\frac{abc}{c}\right] \right\}$$

is there a sequence (with repeats) s.t. top string = bottom string

$$\left[\frac{a}{ab}\right] \left[\frac{b}{ca}\right] \left[\frac{ca}{a}\right] \left[\frac{a}{ab}\right] \left[\frac{abc}{c}\right]$$

a b ca a abc

$$S_1 = \left\{ \left[\frac{abc}{ab}\right], \left[\frac{bc}{a}\right], \left[\frac{cc}{b}\right], \left[\frac{bb}{c}\right] \right\}$$

no solution b/c top string always longer

$$S_2 = \left\{ \left[\frac{abc}{db}\right], \left[\frac{bc}{af}\right], \left[\frac{b}{df}\right] \right\}$$

no solution b/c no d, f in top row

claim: PCP is undecidable

(it is recognizable)

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proof outline:

- prove MPCP undecidable
- show PCP → MPCP

MPCP: like PCP but solution sequence must start with 1st tile in list

To show MPCP undecidable,
simulate a TM

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Suppose M a TM & string w
Make set S of Tiles

a solution for S has top/bottom string

initial config for w

$\# C_1 \# C_2 \# \ldots\ldots \underbrace{\# C_n \#\#}_{\text{almost}}$

configuration of M

abcqdefg

accepting configuration

Make set of tiles S s.t.
$\exists$ solution for S iff
M accepts w

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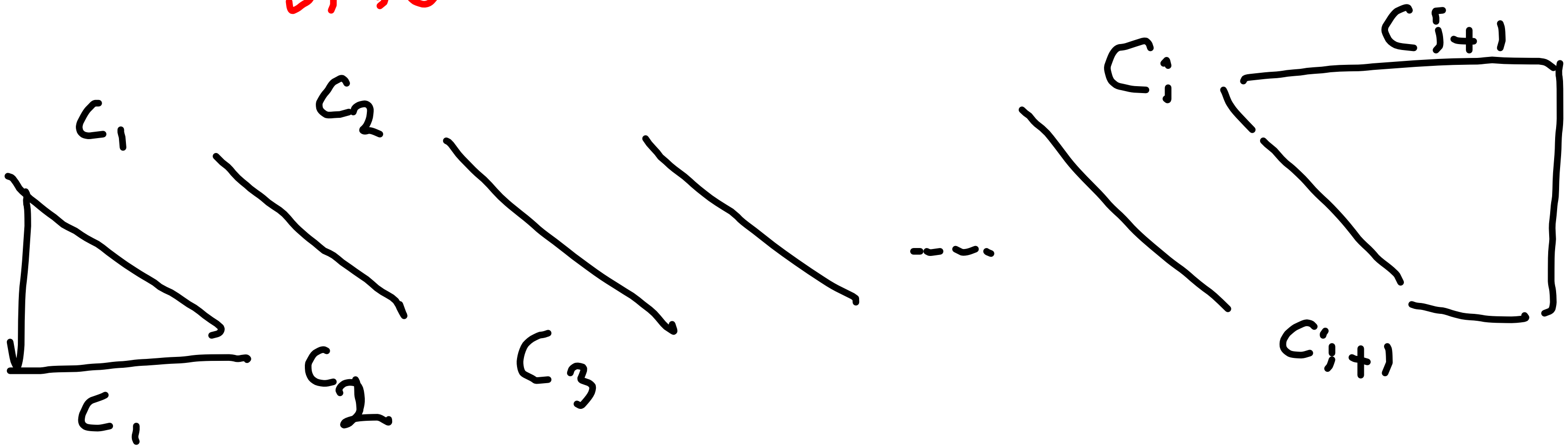
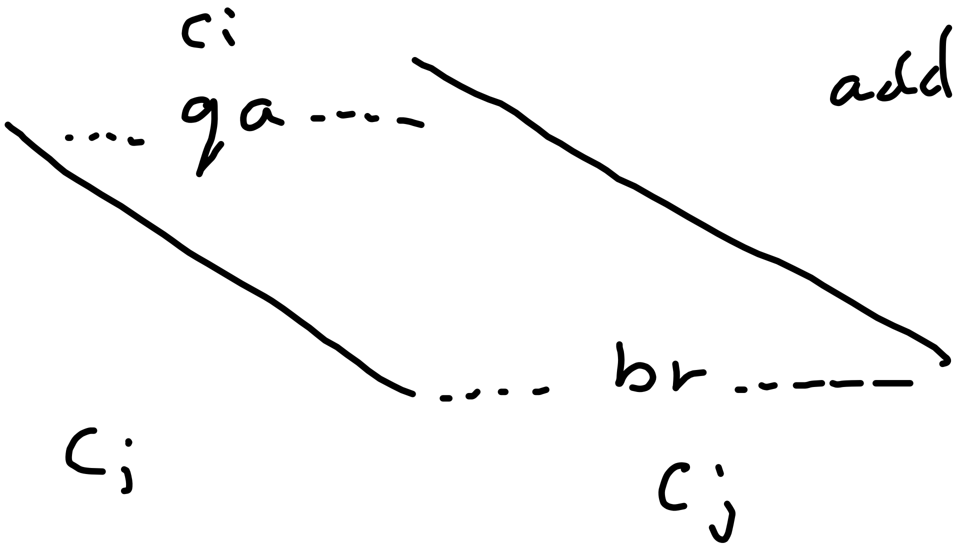
start tile

$$\left[\begin{array}{l} \# \\ \\ \# \; \underbrace{q_0}_{\text{initial state}} \; \underbrace{w_1 \dots w_n}_{\text{input string } w} \; \# \end{array}\right]$$



actions




if $\delta(q,a) = (r,b,R)$

add tile

$$\left[\frac{q\,a}{b\,r}\right]$$

if $\delta(q,a) = (r,b,L)$

add tiles

$$\left[\frac{c\,q\,a}{r\,c\,b}\right] \quad \forall c \in \Gamma$$

# Copying tiles

add to S $\begin{bmatrix} a \\ a \end{bmatrix}$ $\forall a \in \Gamma$

also add to S $\begin{bmatrix} \# \\ \# \end{bmatrix}$ $\begin{bmatrix} \# \\ \sqcup\,\# \end{bmatrix}$

end

$\# \quad \ldots\ldots \quad \# \ldots\ldots\ q_{accept}\ \ldots\ldots,\ \#$

add tile $\begin{bmatrix} q_{accept}\ \# \\ \# \end{bmatrix}$

also add pacman transitions

gobble

$\begin{bmatrix} a\ \ q_{accept} \\ q_{accept} \end{bmatrix} \qquad \begin{bmatrix} q_{accept}\ \ a \\ q_{accept} \end{bmatrix}$

$\forall a \in \Gamma$

If I have a set S of tiles for MPCP problem

Can I build equivalent set of tiles for PCP

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Suppose $w = w_1 w_2 \ldots w_n$

$*w = *w_1 * w_2 \ldots *w_n$

$w* = w_1 * w_2 * \ldots w_n *$

$*w* = *w_1 * w_2 * \ldots * w_n *$

Suppose $S = \left\{ \left[\frac{t_1}{b_1}\right], \left[\frac{t_2}{b_2}\right], \ldots \left[\frac{t_n}{b_n}\right] \right\}$

MPCP

must start sequence

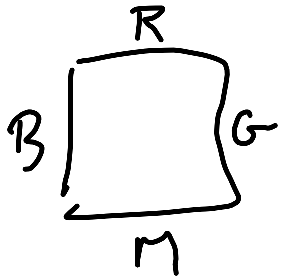
$$S' = \left\{ \left[\frac{*t_1}{*b_1*}\right], \left[\frac{*t_1}{b_1*}\right], \left[\frac{*t_2}{b_2*}\right], \ldots \left[\frac{*t_n}{b_n*}\right], \left[\frac{*\diamond}{\diamond}\right] \right\}$$

Wang tiles

1st aperiodic tiling

1966 Robert Berger

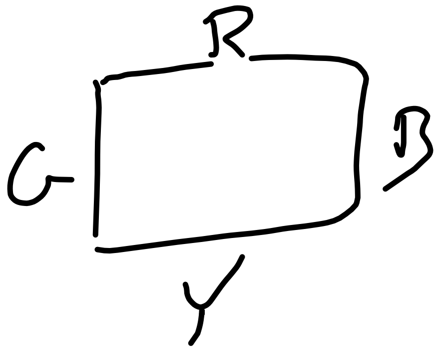
20,426 wang tiles

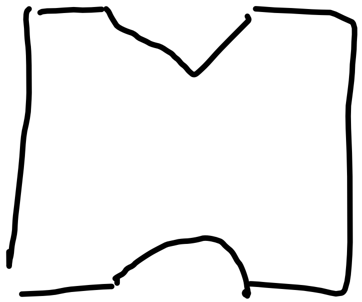


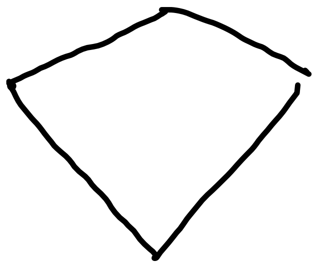

...... 1996

Karel Culik, II

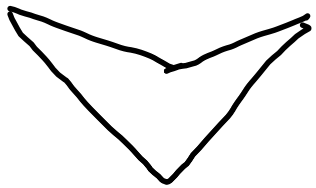
13 wang tiles





kite



dart

Suppose That if a set
of tiles S can
tile whole plane,

it can tile whole
plane periodically.

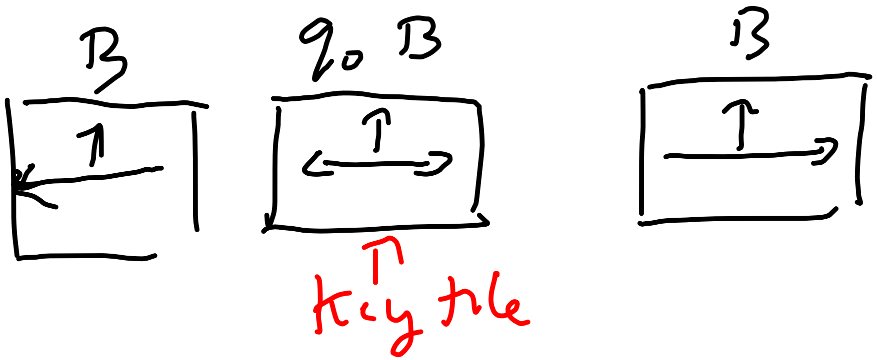
Conjecture by
Hao Wang 1961

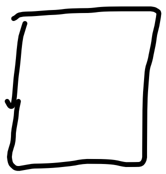
$\Longrightarrow$ WRONG

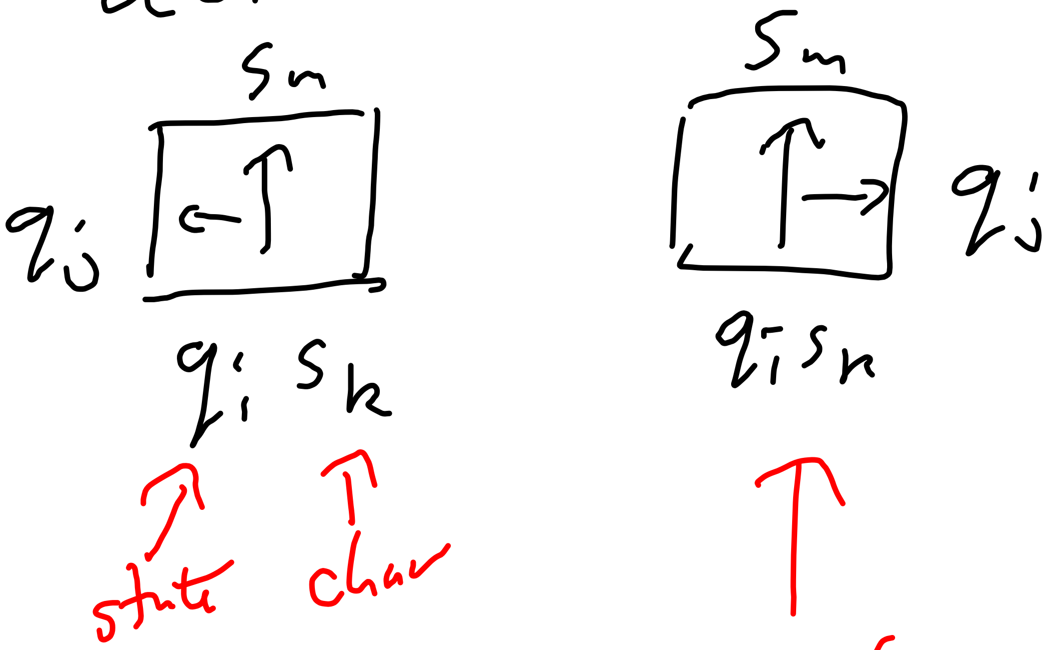
Tiles for "completion problem"

→ given part of tiling, can it be extended to $\mathbb{R}^2$??

start TM on blank input



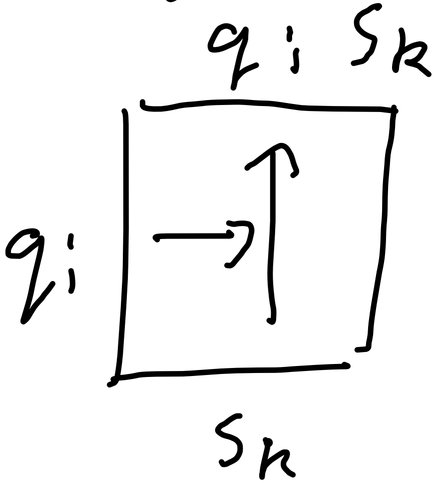

also  bottom 1/2 plane action files

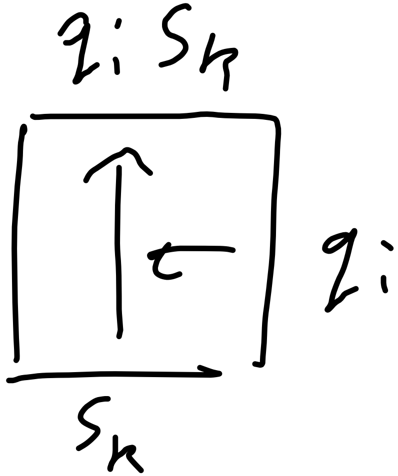



TM has

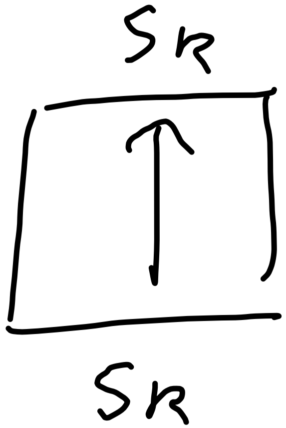
$\delta(q_i, s_k) = (q_j, s_m, R)$

merge tiles




copy tiles



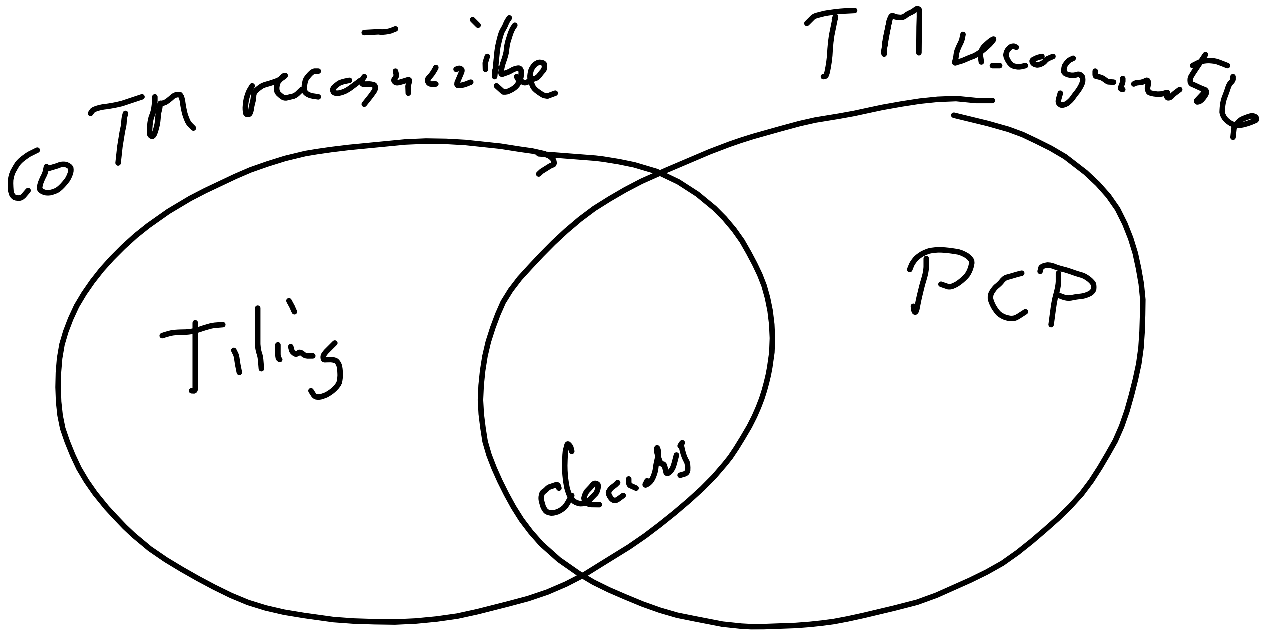
R. Robinson 1977

itsy-bitsy universal TM

(Minsky)

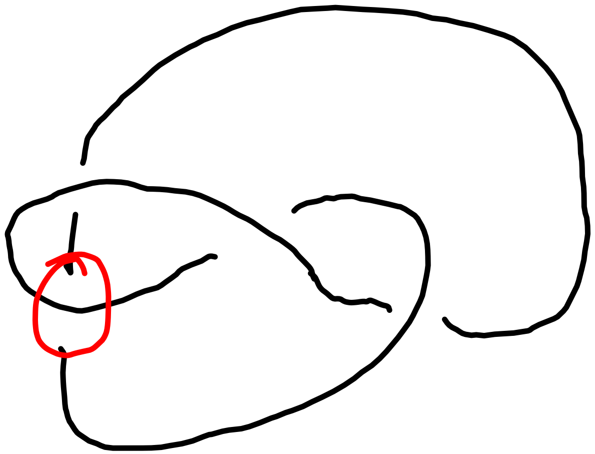
fixed set of 36 tiles

for which completion isn't decidable

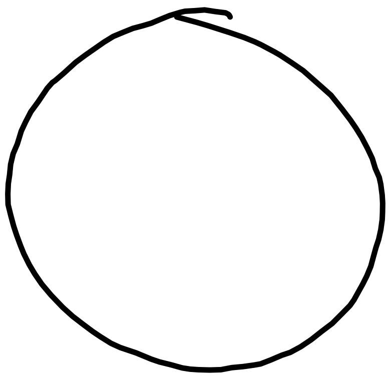

co TM recognizable
TM recognizable
Tiling
PCP
decidable

# Knot diagrams

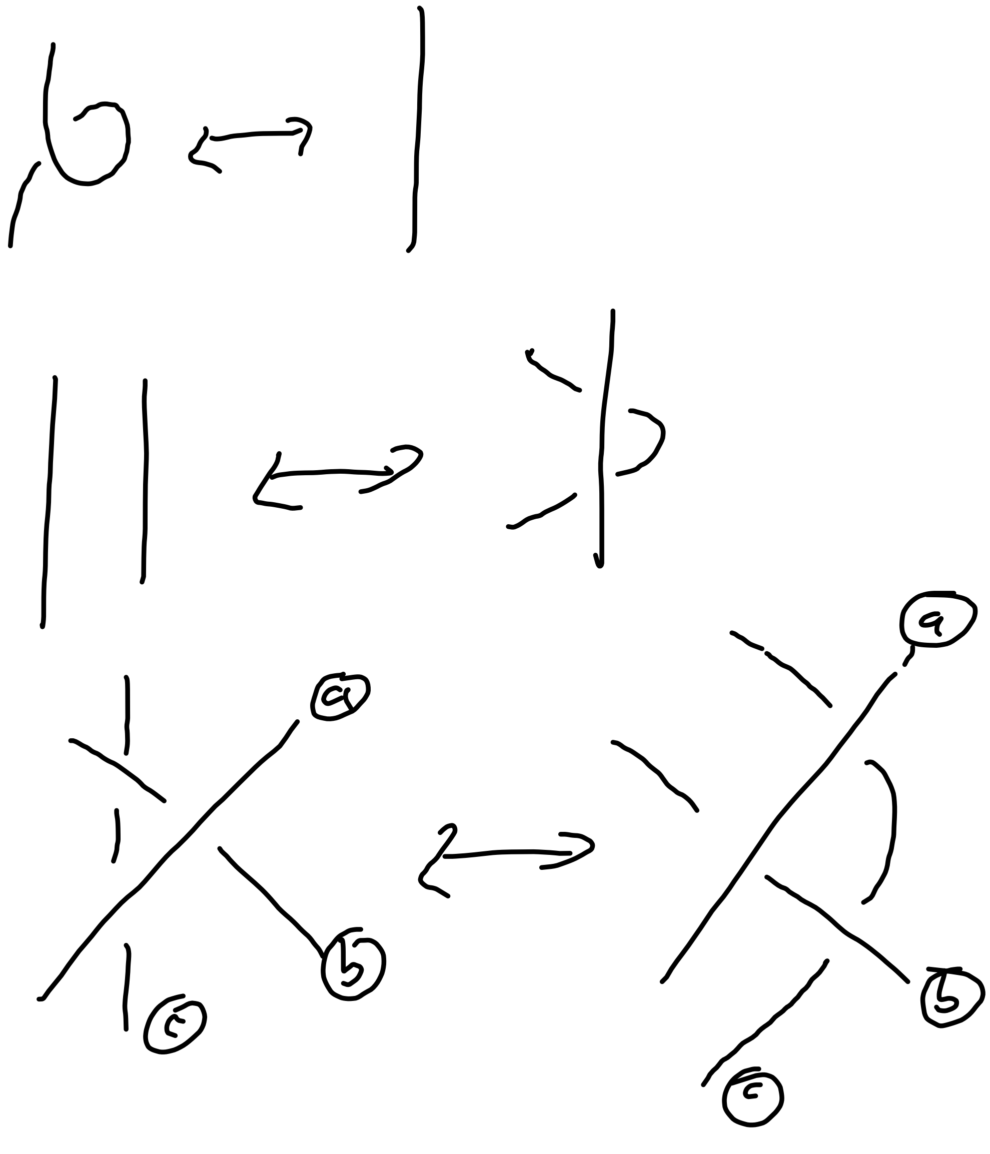
crossing



unknot



# Reidermeister moves



1926/27 R. moves

1934 L. Goeritz

Sometimes must increase
# of crossings

2001 Hass & Lagarias

$n$ crossings

$\leq 2^{cn}$ moves

$c \leq 10^{11}$

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