Miscellaneous topics for regular languages

Exam 7-9pm Thursday

116 RAL

Lecture: review / Q|A

- Regular $\Rightarrow$ PL holds
  but not PL holds $\Rightarrow$ Regular

- Unique minimal DFA

- different varieties of automata

$L = \{a^i b^n c^n : i, n \geq 0\}$

Satisfies PL

$p = 1$

$\forall$ strs w st. $|w| \geq 1$

Let $w = xyz$ where

$x = \varepsilon \quad y = a$

$z = \text{rest} = a^j b^n c^n$

$|xy| \leq 1$ ✓ $\quad |y| \geq 1$ ✓

$w = xyz$ ✓

$xy^i z = a^i a^j b^n c^n$

$L \cap b^* c^*$

$= \{ b^n c^n : n \geq 0 \}$

L is not regular

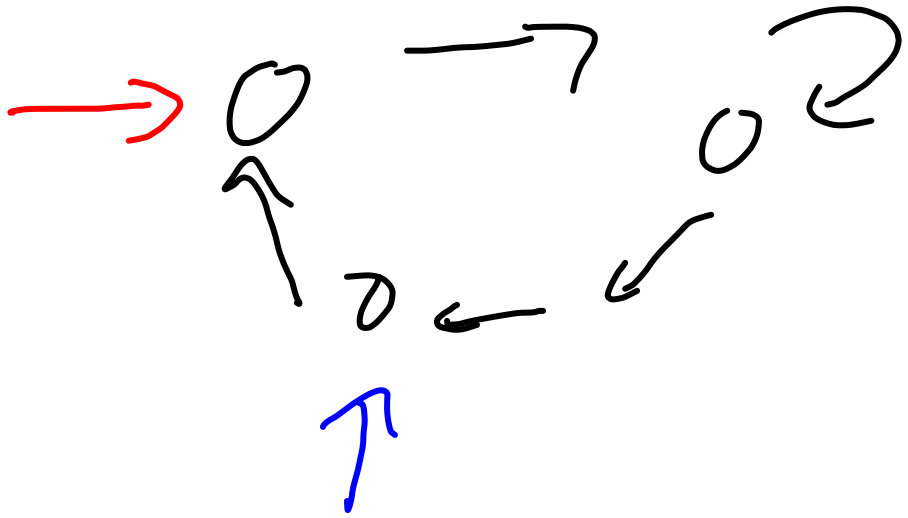
because $b^n c^n$ isn't


S
b

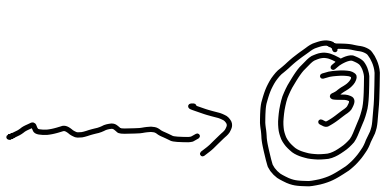
Suffix languages

q a state L recognized by DFA M

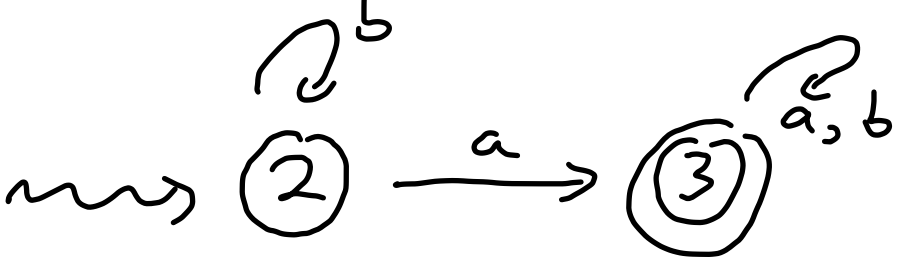
L_q = Language of M if start state changed to q



$$L_q = \{ w : q \xrightarrow{w} q_f, q_f \in F \}$$



$$L_3 = (a+b)^*$$
$$= \Sigma^*$$



$$L_2 = b^* a (a+b)^*$$
$$= L_5$$

Equivalence classes

q, r are states

$q \sim r$ iff $L_q = L_r$

$2 \sim 5$

$3 \sim 6$

$[2] = \{2, 5\}$

$[3] = \{3, 6\}$

$[1] = \{1\}$

Equivalence classes are states of minimal DFA

Minimal DFA depends only on L = set of strings

Let $w \in \Sigma^*$

$$S_L(w) = \{ x : wx \in L \}$$

$$L = (a+bb)\,b^*\,a\,(a+b)^*$$

$$= a\,b^*\,a\,(a+b)^* + bb\,b^*\,a\,(a+b)^*$$

$w = a$

$S_L(w) = b^*\,a\,(a+b)^*$

$w = ab$

$S_L(w) = b^*\,a\,(a+b)^*$

$w = b$

$S_L(w) = b^+\,a\,(a+b)^*$

$=$ not kosher, but grep

$b^+ = bb^*$

$= bb^*\,a\,(a+b)^*$

$w = bb$

$S_L(w) = b^*\,a\,(a+b)^*$

Equivalent strings

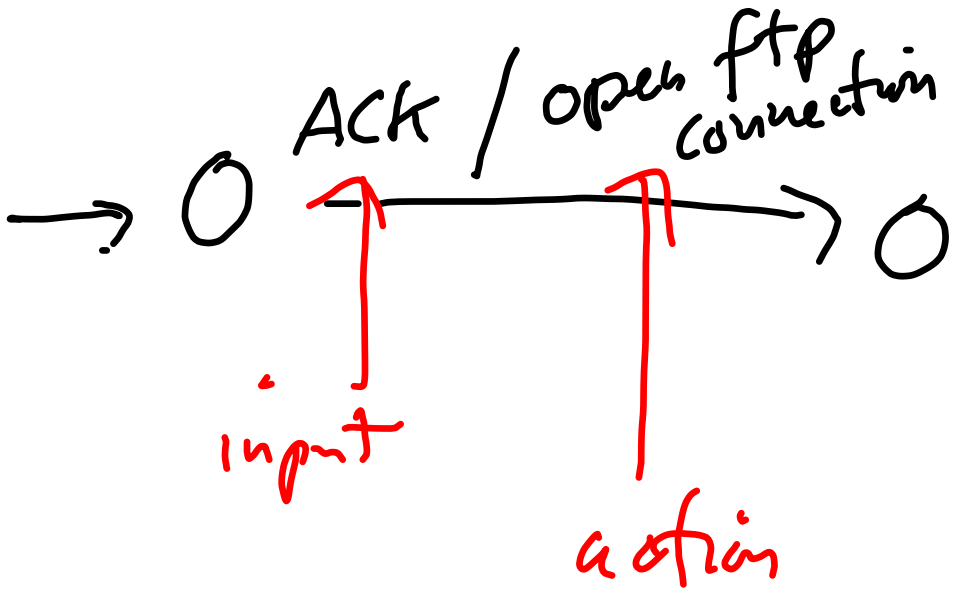
w, z are strings in Σ^*

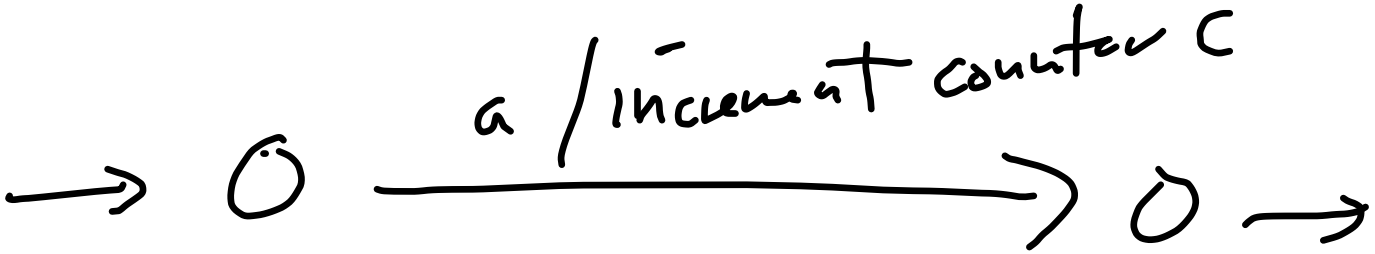
$w \sim z$ iff $S_L(w) = S_L(z)$

Fun Mods to automata

- transducer add output
- probabilities on transitions
- actions on transitions

Actions



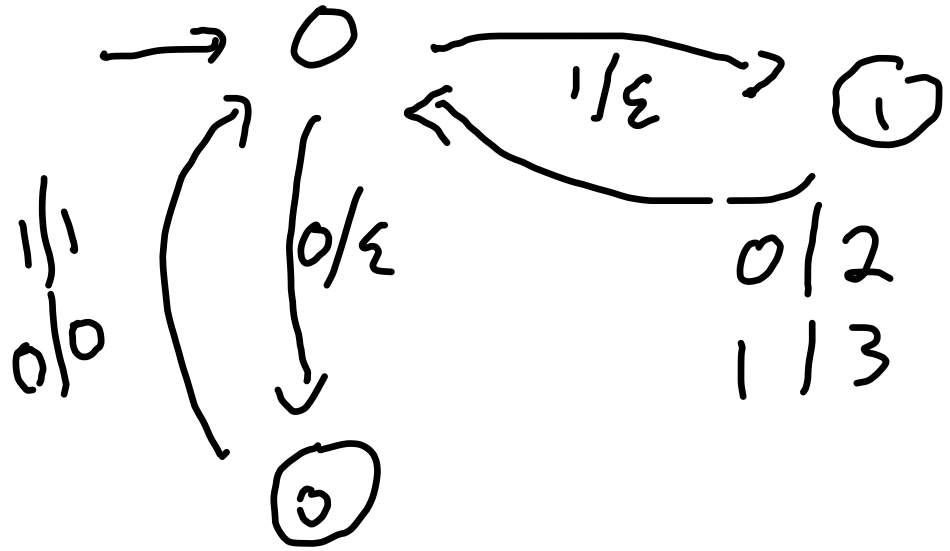


Output

Finite-state Transducers FSTs

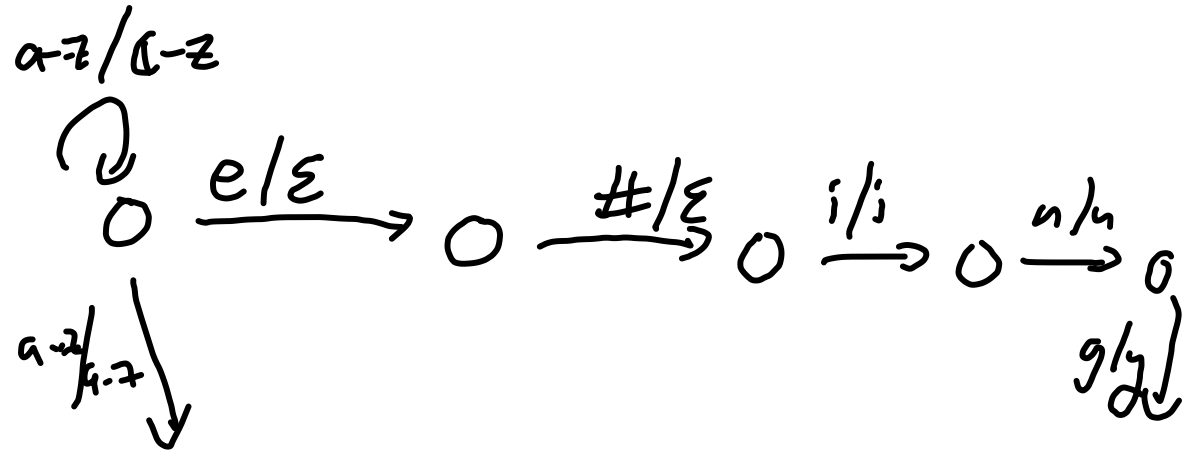
binary #'s into base-4 #'s

01 11 10 ..... → 1 3 2 .....



bake # ing

→ baking

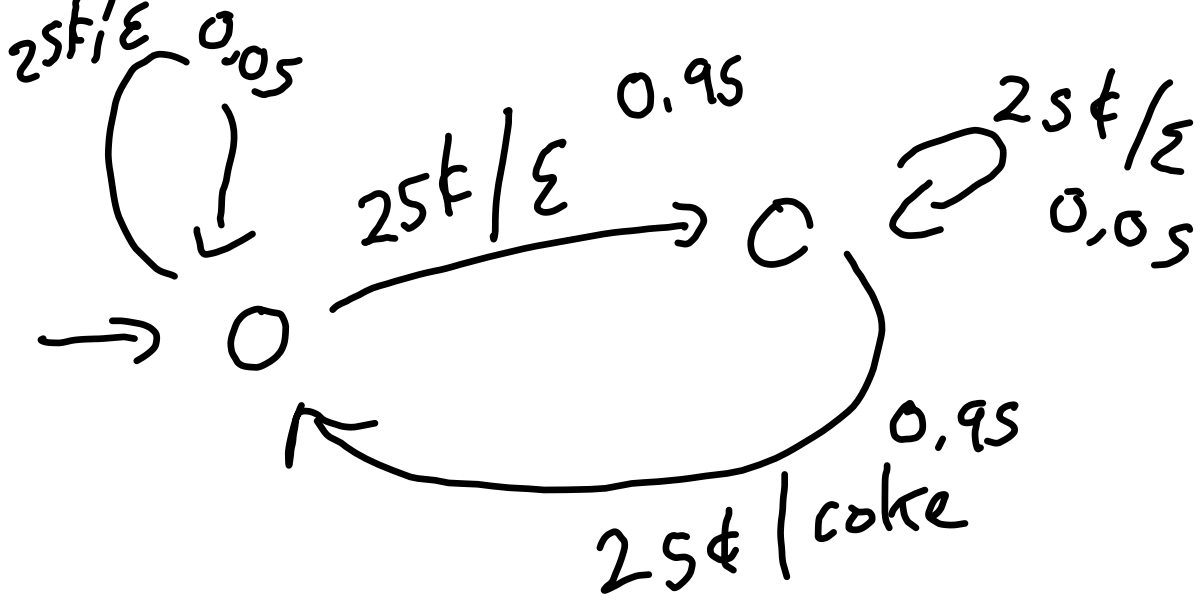


e → ε / ___ # ing

Statistical NFAs / FSTs

add probabilities

Coke machine



Types for δ

GNFAs + above

NFA : $\delta : Q \times \Sigma \to \mathcal{P}(Q)$

FST : Sipser : one char output per input char
deterministic

$\delta : Q \times \Sigma \to Q \times \Gamma$

Γ ↑ output char

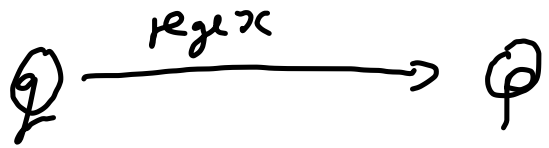
outputs strings on transitions

$\delta : Q \times \Sigma \to Q \times \Gamma^*$

statistical FST

$\delta : Q \times \Sigma \to \mathcal{P}(Q \times \Gamma^* \times [0,1])$

GNFA



$$\delta : Q \times R(\Sigma) \longrightarrow Q$$

$\exists!$ There is exactly one transition from p to q $\forall p, q$ states

$$\delta : Q \times Q \longrightarrow R(\Sigma)$$

source destination regexes over Σ