

CS 273

5/1/2007

May Day!

Last Day!

- CYK algorithm for parsing
- Review of Big Ideas
- Looking forward
- Evaluations

Given a CFG G in CNF form
and a $w \in \Sigma^*$
is $w \in L(G)$?

Ex: C program
is it a valid program?

G - grammar for C

G : in CNF form

$w \in \Sigma^*$

$|w| = n$ G is fixed

CYK -- Cocke-Younger-Kasami

$O(n^3)$ time algorithm

$O(g n^3)$

↑ size of grammar

Valiant reduced this problem
to boolean matrix multiplication

To multiply two $n \times n$ matrices
there is an $O(n^{2.343\dots})$ time
algorithm.

Given h, w
is $w \in L(h)$

$$w \in L(h) \Leftrightarrow S \stackrel{*}{\Rightarrow} w$$

$$S \Rightarrow AB \stackrel{*}{\Rightarrow} w$$

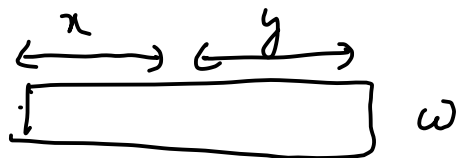
$$A \stackrel{*}{\Rightarrow} x$$

$$B \stackrel{*}{\Rightarrow} y$$

$$w = xy$$

$$|x| < |w|$$

$$|y| < |w|$$



to check $S \stackrel{*}{\Rightarrow} w$

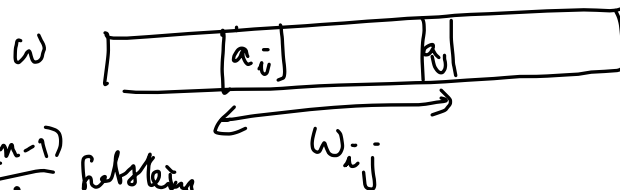
need to know how substrings of
 w are derived by other variables

algorithmic idea
— dynamic programming

a way to make recursive algorithms efficient by re-using work. (via storing)

$$\omega = a_1 a_2 a_3 \dots a_n$$
$$a_i \in \Sigma \quad i=1, 2, \dots, n$$

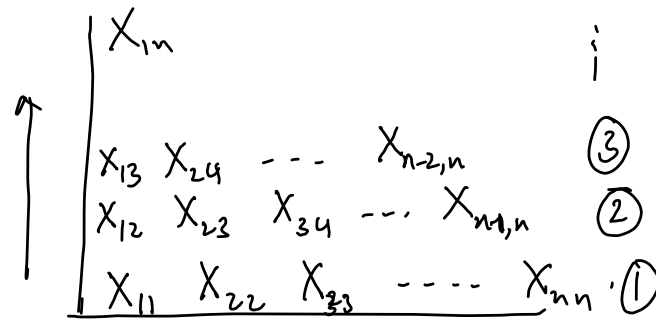
$$\omega_{ij} = a_i a_{i+1} \dots a_j$$



$$\binom{n}{2} = \frac{n(n-1)}{2} \text{ subseqs}$$

$$X_{ij} = \{ A \in V \mid A \stackrel{*}{\Rightarrow} \omega_{ij} \}$$

$$X_{ii} = \{ A \mid A \stackrel{*}{\Rightarrow} a_i \}$$



$$\omega \in L(G) \Leftrightarrow S \xRightarrow{*} \omega$$

$$\Leftrightarrow S \in X_{1n}$$

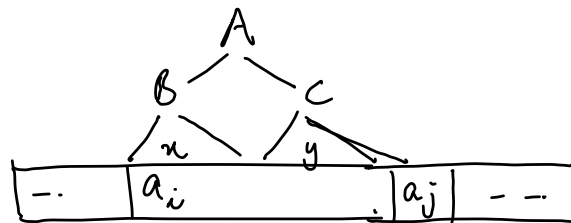
$$X_{ij} = \{A \mid A \xRightarrow{*} w_{ij}\}$$

$A \in X_{ij}$ iff $\exists k, B, C$
 s.t. $(j-i \geq 1)$

① $A \rightarrow BC$ is a rule in G

② $B \in X_{ik}$

③ $C \in X_{k+1,j}$



Algorithm

1. for $i = 1$ to n
 Compute X_{ii} using base case
2. for $l = 1$ to $n-1$ do // increasing
 substring
 length
 for $i = 1$ to $n-l$ do
 $j = i + l$
 $X_{ij} = \emptyset$
 for $k = i$ to $j-1$ do
 $X_{ij} = X_{ij} \cup \{ A \mid$
 $A \rightarrow BC \text{ is a rule}$
 $B \in X_{ik}$
 $C \in X_{k+1, j} \}$

Review of Big Ideas

- ① Computation is a formal object that can / should be reasoned about
- ② Non-determinism
(a mathematical construct)
(P vs NP)
- ③ Equivalence of different models via simulation
(Specification vs Verification
logic vs Computation)
- ④ Formal defn of algorithm as a TM
- ⑤ Consequences of Alg \Leftrightarrow TM
 - undecidability of useful tasks
 - formal way to reason about computational complexity
- ⑥ Reductions

Looking Forward

