CS 273 2/15/07

Announcements

1. HW 4 due date extended
   to Mon, 19th 3-30 pm

2. HBS make up.
   Today (Thur) 5-6 pm, 0220 SC
   Tomorrow (Fri) 3-4 pm ??

3. Midterm, 22nd Feb, 7-9 PM
   Roger Adams Lab, Room 116

4. Late homeworks ??
   3.30 pm is sharp deadline.

Proving Languages Non-Regular

- Direct methods
- Pumping Lemma
- closure properties (reductions)

$L = \{0^n 1^n \mid n \geq 1\}$
$= \{01, 0011, 000111, \ldots\}$

"limited memory"

"read once"

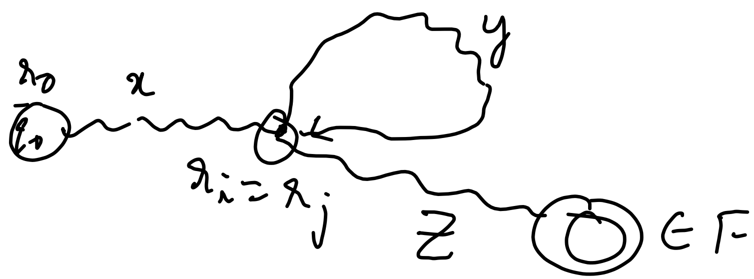
Suppose $\exists M$ that recognizes L.

Let p be # of states of M

Consider $w = 0^p 1^p \in L$

$\underbrace{r_0 r_1 r_2 r_3 \ldots r_p}_{\text{first } p+1 \text{ states}} \ldots r_{2p}$

Some r_i and r_j, $0 \le i < j \le p$
are same state q of M



x, y contain only 0's

Consider string $w' = xz$ is
$\Rightarrow$ also accepted
but xz is not in L!
$\Rightarrow$ a contradiction

Pumping Lemma:

Let L be a regular language.
Then there exists an integer p
such that for any string $w \in L$
with $|w| \geq p$, w can be written
as xyz with following properties

1. $|xy| \leq p$
2. $|y| \geq 1$
3. $xy^iz \in L \quad \forall i \geq 0$

Proof: L is regular

$\Rightarrow \exists$ DFA M for L.

Let p be # of states of M

We use p for lemma.

let $w \in L$, s.t $|w| \geq p$ $=n$

let $r_0 r_1, r_2 \ldots r_n$ be the sequence of states that M goes into on reading w.

(1) $r_0 = q_0$ — start state

(2) $\delta(r_{i-1}, w_i) = w_{i+1}$ $\quad i = 1, 2, \ldots$

(3) $r_n \in F$

Consider $r_0 \, r_1 \cdots r_p \cdots r_n$

$\underleftrightarrow{\quad r_0 \, r_1 \cdots r_p \quad}$ $p+1$ states

From pigeon hole principle

$\exists \, l, k$ s.t $0 \le l < k \le p$

and $r_l = r_k$

$x = w_1 w_2 \cdots w_l$, $y = w_{l+1} \cdots w_k$

$z = w_{k+1} \cdots w_n$

① $|xy| \le p$

② $|y| \geq 1$ ③ $xy^i z \in L \ \forall i \geq 0$

Use pumping lemma to prove $L = \{0^n 1^n \mid n \geq 1\}$ is not regular.

Suppose L is regular

then pumping lemma applies!

$\Rightarrow \exists$ an integer p ....

$\forall$ string w s.t. $|w| \geq p$, $w \in L$

lets choose $w = 0^p 1^p \in L$

$|w| = 2p \geq p$.

By Lemma

$w = 0^p 1^p = xyz$ s.t

① $|xy| \le p$ ② $|y| \ge 1$

③ $xy^i z \in L$ $\forall i \ge 0$

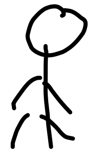
$x = 0^k$ $y = 0^l$ $k + l \le p$

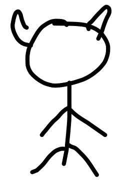
$l \ge 1$

$z = 0^{p-k-l} 1^p$

from ③ $xy^0 z = xz \in L$

but $xz = 0^{p-l} 1^p \notin L$





L is not regular | L is regular

?

01

How many states does your M have?

~~L~~ p

Choose $w \in L$ $0^p 1^p$

By PL $w = xyz$

x, y satisfy PL

Choose i

xy^iz

$xy^iz \notin L$

Victory!!!

$$L = \{ 0^{n^2} \mid n \geq 1 \}$$

$$= \{ 0, 0000, \underbrace{000 \cdots 0}_{9}, \cdots \}$$

Assume L is regular

By PL $\exists$ p .....

$w = 0^{p^2}$ ① $w \in L$
② $|w| \geq p$

$w = xyz$

ⓐ $\cancel{|xy|}$ $|xy| \leq p$ ⓑ $|y| \geq 1$

$x = 0^k \quad y = 0^l$

$k + l \leq p, \quad l \geq 1$

Choose i??

$i = 2$

$$xy^2z = 0^k \cdot 0^{2l} \cdot 0^{p^2 - l - k}$$

$$= 0^{p^2 + l}$$

If $0^{p^2+l} \in L$

$\Rightarrow \quad p^2 + l = j^2$ for some j

$$p^2 < p^2 + l \leq p^2 + p < (p+1)^2$$

$\Rightarrow \quad p^2 + l \neq j^2$

and hence $xy^2z \notin L$

$$L = \{ 0^p \mid p \text{ is prime} \}$$

$L = \{0^n 1^n \mid n \geq 1\}$ is not regular.

$L' = \{w \in \{0,1\}^* \mid w$ has equal # of 0's and 1's$\}$

$L = L' \cap 0^* 1^*$

$\therefore$ Suppose L' is regular

then since $0^* 1^*$ is regular

L is also regular,

(closure under intersection)

Title: Feb 15-12:13 PM (15 of 16)

$L'' = \{ w \mid w$ has different # of 0's and 1's$\}$

$L' = \overline{L''}$

Suppose L'' is regular

then L' is regular because regular langs are closed under Complementation.

But L' is not regular!

Title: Feb 15-12:13 PM (16 of 16)