# CS 273, Fall 2006
# Exam 2
# 2 November 2006

**INSTRUCTIONS (read carefully)**

- Fill in the following information giving name and ID.

**NAME:**

**NETID:**

- CLEARLY print your name and ID on every page.
- There are 7 problems, on pages numbered 2 through 9, plus a blank page numbered 10. Make sure you have a complete exam.
- The point value of each problem is indicated next to the problem, and below.
- The exam is designed for slightly over one hour, although you have two hours.
- It is probably wise to glance at all problems and point values before beginning, to best plan your time.
- This is a closed book exam. No notes of any kind are allowed. Do all work in the space provided, using the backs of sheets if necessary. See the proctor if you need more paper.

| Problem | Possible | Score |
|---|---|---|
| 1 | 9 | |
| 2 | 9 | |
| 3 | 8 | |
| 4 | 8 | |
| 5 | 8 | |
| 6 | 10 | |
| 7 | 8 | |
| Total | 60 | |

## Problem 1: True/False (9 points)

Completely write out "True" if the statement is necessarily true. Otherwise, completely write "False". For example, "$x + y > x$" has the answer "False" assuming that $y$ could be 0 or negative. But "If $x$ and $y$ are natural numbers, then $x + y \geq x$" has the answer "True". You do not need to explain or prove your answers.

1. If a grammar $G$ is in Chomsky-normal form, then each word in $L(G)$ has exactly one parse tree.

2. If a language $L$ satisfies the conditions of the pumping lemma for regular languages, then $L$ is regular.

3. Let $\Sigma = \{0, 1\}$. Let $L = \{0^n 1^n 0^m 1^m : n, m \geq 0\}$. $L$ is context-free.

4. The context-free languages are closed under intersection.

5. The language $L = \{a^n a^n : n \geq 0\}$ is not regular.

6. The deterministic and non-deterministic versions of PDA's are equivalent.

7. The language $L = \{a^n b^n c^n : a, b, c \geq 0 \text{ and } a, b, c \leq 1000\}$ is context-free.

8. Let $\Sigma = \{a, b, c\}$. The language $\{w\#w : w \in \Sigma^*\}$ is context-free.

9. Let $\Sigma = \{a, b\}$. The language described by the regular expression $a^*b^*$ is a context-free language.

## Problem 2: Short answer (9 points)

The following questions require only short answers.

1. Given two context-free grammars $G_1$ and $G_2$, with start symbols $S_1$ and $S_2$, explain precisely how to construct a grammar $G$ such that $L(G) = L(G_1) \circ L(G_2)$. (Recall that $A \circ B$ is the concatenation of the two languages $A$ and $B$.)

2. What is the difference between a Turing recognizable language and a Turing decidable language?

3. Let $G$ be the following context-free grammar, where lowercase letters are terminals (i.e. $\Sigma = \{a, b\}$), uppercase letters are variables, $S$ is the start symbol.

   $$S \Rightarrow SS \mid (X) \mid \epsilon$$
   $$X \Rightarrow XX \mid (X) \mid a \mid b \mid \epsilon$$

   Give a precise, non-recursive definition of $L(G)$.

## Problem 3: Grammar construction (8 points)

Let $\Sigma = \{a, b, c\}$. Give a grammar that generates the language $L = \{a^i b^i c : i \geq 0\} \cup \{ab^i c^i : i \geq 0\}$. Carefully explain how your grammar works, and what each nonterminal does. (You don't need to give a formal proof that it does the right thing.)

## Problem 4: PDA Building (8 points)

Let $\Sigma = \{0, 1\}$. Give a PDA for $L = \{x\#y^R : x, y \in \Sigma^* \text{ and } y \text{ is a prefix of } x\}$. Notice that $y^R$ is the reversed version of the string $y$ and that $y$ is a prefix of $x$ if there is some string $w$ such that $x = yw$. Also, $x$ and $y$ can be equal: every string is a prefix of itself.

Present your PDA as a state diagram. Carefully explain how your PDA works, and the meaning of each state and/or transition. (You don't need to give a formal proof that it does the right thing.)

## Problem 5: Pumping lemma (8 points)

Let $\Sigma = \{a, b\}$. Let $L = \{wbbw : w \in \Sigma^*\}$. Prove that $L$ is not regular by filling in the missing parts of the following pumping lemma proof.

Suppose that $L$ were regular. Let $p$ be the constant given by the pumping lemma.

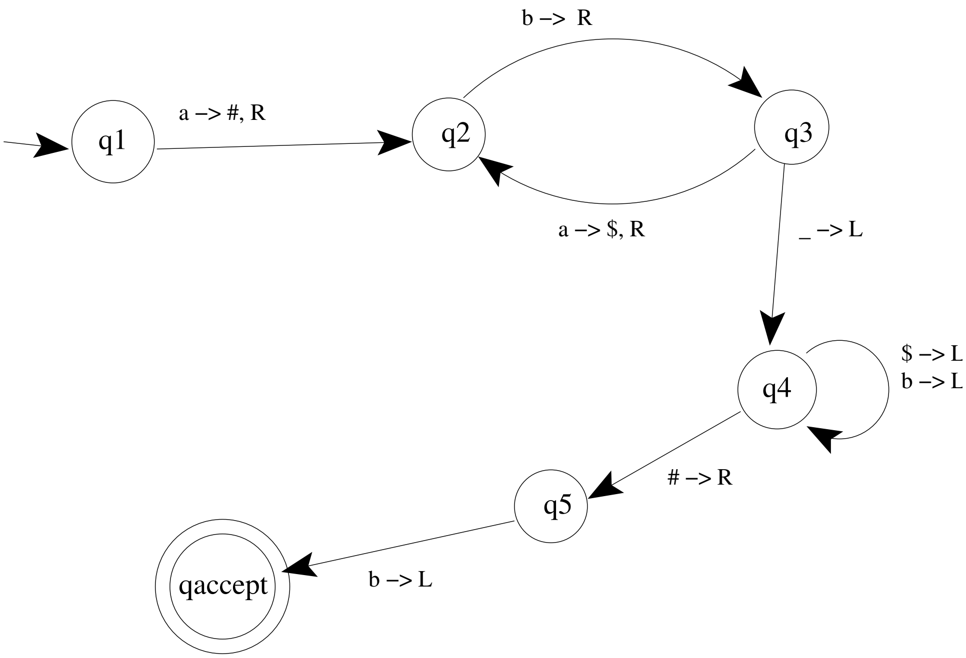
Consider the string $w_p =$

Because $|w_p| \geq p$, there must exist strings $x$, $y$, and $z$ such that $w_p = xyz$, $|xy| \leq p$, $|y| > 0$, and $xy^iz \in L$ for every $i \geq 0$.

Since                                  isn't in $L$, we have a contradiction. Therefore, $L$ must not have been regular.

## Problem 6: Turing machines (10 points)

Consider the following state diagram for a Turing Machine $M$. The input alphabet is $\{a, b\}$ and the diagram uses underscore to represent the special blank character.




(a) Give a run on the input string $abab$. That is, show the sequence of configurations starting with $q_0abab$ and ending with an accepting configuration.

(b) What language does $M$ recognize?

(c) Describe the pattern of how $M$ moves back-and-forth along the tape.

(d) Where does $M$ leave the tape head when it halts in the accept state?

(e) Describe how $M$ modifies the contents of its tape. That is, what is the difference between the original tape contents and the tape contents when the Turing Machine halts in the accept state?

## Problem 7: PDA modification (8 points)

If $L$ is a context-free language and $L'$ is a regular language, then $L \cap L'$ is context-free. To show this, let $M = (Q, \Sigma, \Gamma, \delta_M, q_0, F_M)$ be a PDA recognizing $L$ and let $N = (P, \Sigma, \delta_N, p_0, F_N)$ be a DFA recognizing $L'$. We then need to construct a PDA $M'$ recognizing $L \cap L'$.

To do this, set $M' = (Q \times P, \Sigma, \Gamma, \delta', (q_0, p_0), F')$.

(a) Express $F'$ in terms of $F_M$ and $F_N$.

(b) If $c$ is a character in $\Sigma$ (i.e. $c$ is not $\epsilon$), $(q, p)$ is a pair of states from $Q \times P$, and $t$ is a stack symbol in $\Gamma_\epsilon$, what is the value of $\delta'((q, p), c, t)$?

(c) If $(q, p)$ is a pair of states from $Q \times P$ and $t$ is a stack symbol in $\Gamma_\epsilon$, what is the value of $\delta'((q, p), \epsilon, t)$?

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