Announcements

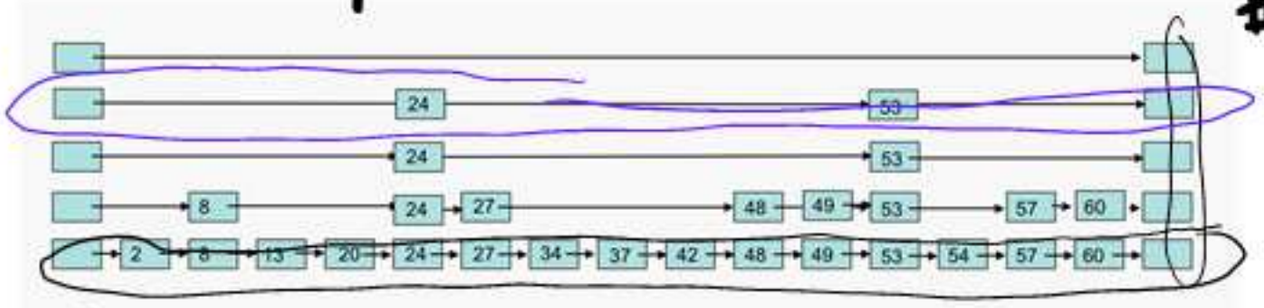
Exam II 11/9 7-9p email conflicts

Material Through today

Today:

Skiplist Analysis

Analysis of SkipLists – Space



pointers in any ~~row~~ level i corresponds to # of nodes reaching level $i-1$.

Facts:

① Prob that i "tails" in a row occur is $\left(\frac{1}{2}\right)^i$.
this is prob that a node exists at level i.

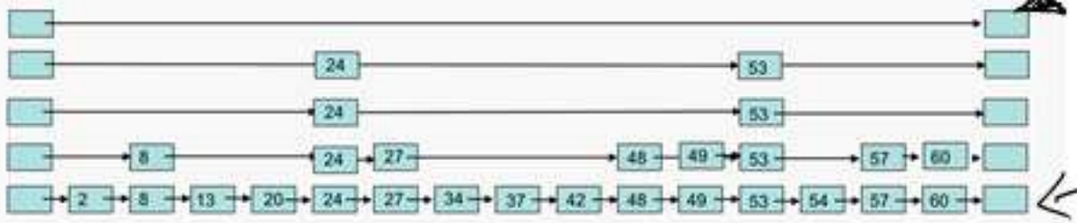
② If each of n "trials" "items" is present in a set w. probability p "success" then the expected # of items in the set is np. Binomial Dist. (random variable counting # of successes.)

$$E[\text{total \# of pointers}] = E\left[\sum_{i=0}^{\infty} \#\text{ of pointers in row } i\right] = \sum_{i=0}^{\infty} E[\#\text{ of pointers at level } i]$$

$$= \sum_{i=0}^{\infty} n \cdot \left(\frac{1}{2}\right)^i = 2n = O(n)$$

Analysis - Height

With high probability the height is $\leq O(\log n)$ (n items)



Fact ③ If each of n events has prob p of occurring, the probability that 1 or more occur is $1-(1-p)^n \leq n \cdot p$ (Markov's Inequality)

$$1-\left(\binom{n}{0}1^{n}(-p)^{0}+\binom{n}{1}1^{n-1}(-p)^{1}+\binom{n}{2}1^{n-2}(-p)^{2}\cdots\right)$$

$$1-\left(1+np-\binom{n}{2}p^{2}+\binom{n}{3}p^{3}-\cdots\right) \leq np$$

→ Seq of decreasing terms

① any one node reaches level i w.p. $\left(\frac{1}{2}\right)^i$

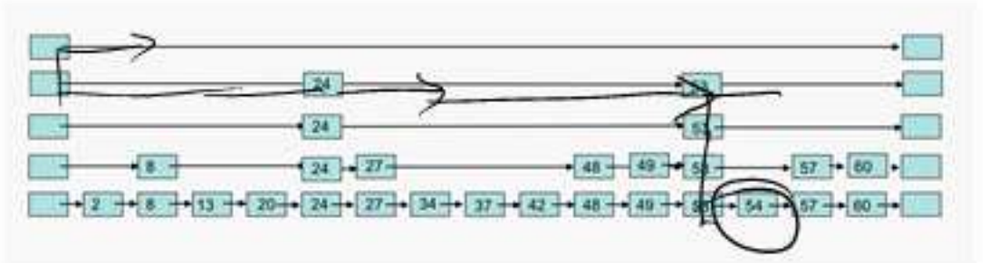
Prob there is any node at level i is $\leq n\left(\frac{1}{2}\right)^i$

$i = 3\log n$

prob level $3\log n$ has elts is

$$\leq \frac{n}{2^{3\log n}} = \frac{1}{n^2}$$

Analysis – Search



SkipSearch

Scans forward $O(\log n)$ over $O(\log n)$ levels

Scans Down $O(\log n)$

w.h.p.

# of scans = scans forward + scans down

= $O(\log n)$ with high probability

worst case is beyond comprehension

but it won't happen.

Summary:

Implementation	Search Time	Insertion Time	Deletion Time
Skip lists	0.051 msec (1.0)	0.065 msec (1.0)	0.059 msec (1.0)
non-recursive AVL trees	0.046 msec (0.91)	0.10 msec (1.55)	0.085 msec (1.46)
recursive 2–3 trees	0.054 msec (1.05)	0.21 msec (3.2)	0.21 msec (3.65)
Self–adjusting trees:			
top-down splaying	0.15 msec (3.0)	0.16 msec (2.5)	0.18 msec (3.1)
bottom-up splaying	0.49 msec (9.6)	0.51 msec (7.8)	0.53 msec (9.0)

Table 2 - Timings of implementations of different algorithms