

# CS 473UG: Combinatorial Algorithms, Spring 2006

## Homework 8

Due Tuesday, April 18, 2006, at the beginning of class

1. Do problem 10 on pages 508-509 of the textbook.
2. Two problems  $A$  and  $B$  are *polynomially equivalent* if  $A \leq_p B$  and  $B \leq_p A$ . Show that Knapsack (described below) and Subset Sum (see page 491 of the textbook) are polynomially equivalent.

### Knapsack

Instance: A finite set  $U$ , a value  $v(u) \in \mathbb{Z}^+$  for each  $u \in U$ , and a value goal  $K \in \mathbb{Z}^+$ .

Question: Is there a subset  $U' \subseteq U$  such that

$$\sum_{u \in U'} v(u) = K$$

3. Prove that the following problems are all polynomial-time equivalent, that is, if *any* of these problems can be solved in polynomial time, then *all* of them can.
  - K-COLORING: Given a graph  $G$  and an integer  $k$ , does there exist a valid coloring of the vertices of  $G$  using at most  $k$  colors?
  - FINDKCOLORING: Given a graph  $G$  and an integer  $k$ , find a valid coloring of the vertices of  $G$  using at most  $k$  colors.
  - MINCOLORING: Given a graph  $G$ , find the minimum number of colors with which  $G$  can be colored.
  - FINDMINCOLORING: Given a graph  $G$ , find a valid coloring using as few colors as possible.