Homework Problem Set 1
CS 473UG – Spring 2006

Assigned On: January 31, 2006
Due On: February 7, 2006

Instructions: Solve these problems in groups of size at most 3. Solution to each problem must be on a separate sheet; this assists us greatly in grading the problems.

Recommended Reading: Chapter 4 of the textbook and lecture notes on the web.

Problem 1 Solve problem 4.26 from the class textbook.

Problem 2 Consider a data structure for Union-Find that performs path compression in `find` as discussed in class. Starting from `makeUnionFind`(S), show that any sequence of m `union` and `find` operations, where all the `union` operations appear before any of the `find` operations, takes $O(m)$ time.

Problem 3 As observed in section 4.8 of the textbook, there are situations where we would like to modify the code for an alphabet, and communicate the new code over a channel. This problem is about how one can represent a Huffman Code as a bit string.

Assume that the alphabet is $S = \{0, 1, 2, \ldots n-1\}$. Show that the optimal prefix code for alphabet S can be represented using $2n - 1 + n\lceil \log n \rceil$ bits. *Note:* It is possible that a symbol in S is encoded using $O(n)$ bits in the optimal prefix code. (Can you think of an example?) So just sending the code of each symbol will not be efficient.