CS 418: Homework #2

Assigned: Tuesday February 28, 2006

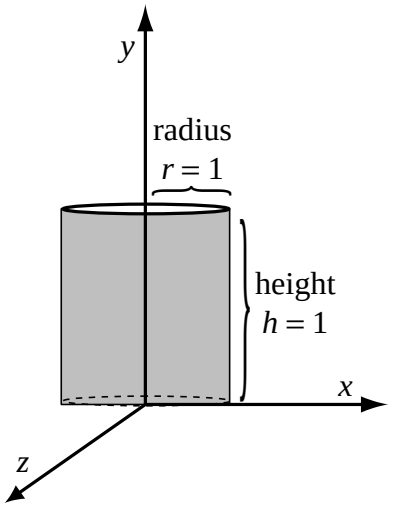
Due: Tuesday March 7, 2006 at the beginning of class

Please be organized when writing your answers to these questions. Make sure that all solutions are clearly indicated and labelled with the question they are answering. Remember to write clearly and legibly. *Unreadable answers will receive 0 credit.*

1. (20pt) You are given two quaternions, $\mathbf{q_1} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\mathbf{k}$ and $\mathbf{q_2} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.
   (a) What is the norm of $\mathbf{q_1}$?
   (b) What is the inverse of $\mathbf{q_1}$?
   (c) Compute $\mathbf{q_1 q_2}$ and $\mathbf{q_2 q_1}$. Are they equal ?
   (d) $\mathbf{q_1}$ represents a rotation. What are the rotation axis and angle ?
2. (10pt) Suppose you are given the 4×4 transformation matrix

$$\mathbf{M} = \begin{bmatrix} \cos\theta & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & \cos\theta \end{bmatrix}$$

   For a given value of θ, describe the geometric effect of applying this transformation to a torus. (You may assume that $\cos\theta \neq 0$.)
3. (20pt) Harvey has written some code to draw the following unit cylinder:



   His drawing code places vertices along the upper and lower rims of the cylinder, connecting them together with triangles which stretch the entire length of the cylinder. He draws the cylinder using a shiny material. Given the position of the light, he expects to see a specular highlight in the middle of the cylinder.
   (a) When Harvey executes his OpenGL code (using Gouraud shading) no highlight appears in the middle of the cylinder. However, when he renders it with RenderMan (using Phong shading) the highlight appears. Explain why this is.

(b) How could the OpenGL code be altered (still using Gouraud shading) so that the highlight would appear?

4. (10pt) Suppose we use a pinhole camera to take a picture of a real rectangular carpet, and the picture covers the entire carpet. Note that the carpet may become a quadrilateral in the picture because of perspective projection. The resolution of the picture is 640x480. The pixel locations of the four corners of the carpet in the picture are $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$. Later we produce a 3D virtual model of the carpet as a rectangle in the world coordinate system and render it using texture mapping in OpenGL. We would like to use the picture as the texture image.

   (a) How can we obtain the texture coordinates for the corners of the carpet so that the texture-mapped carpet looks like the original one ?

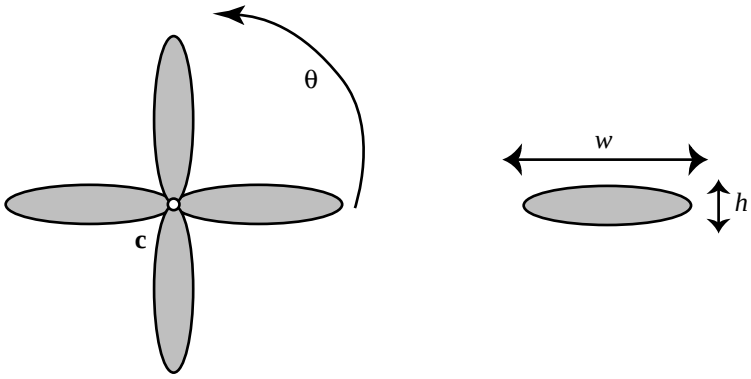
   (b) Is it possible that the edges of the carpet become curved in the picture ? Why ?

5. (20pt) In class we derived the basis matrices for cubic Bézier curves and Hermite curves. Cubic B-splines are another type of piecewise smooth curves. Each segment of a cubic B-spline curve is defined by four consecutive control points, $\mathbf{p}_{i-3}, \mathbf{p}_{i-2}, \mathbf{p}_{i-1}$, and, $\mathbf{p}_i$, as follows,

$$\mathbf{p}(u) = b_1(u)\mathbf{p}_{i-3} + b_2(u)\mathbf{p}_{i-2} + b_3(u)\mathbf{p}_{i-1} + b_4(u)\mathbf{p}_i,$$

   where $b_1(u) = \frac{1}{6}(1-u)^3$, $b_2(u) = \frac{1}{6}(3u^3 - 6u^2 + 4)$, $b_3(u) = \frac{1}{6}(-3u^3 + 3u^2 + 3u + 1)$, and $b_4(u) = \frac{1}{6}u^3$. Derive the basis matrix for the B-spline formulation.

6. (20pt) Suppose that we are building a model of a propellor-driven airplane that we can use in an exciting new animation. Each propellor will be modelled as a *planar* object made out of ellipses, as shown below:



   You have decided to provide two "control knobs" to the animator using this model: the center of the propellor ($\mathbf{c}$) and the angle by which the blades are rotated about this center (θ). Each of the four blades is an ellipse, as shown on the right.

   (a) Construct a transformation hierarchy for the object pictured above. The only geometric primitive you may use is a circle of radius 1 centered about the origin. Your hierarchy may only contain transformation nodes and geometry nodes that draw unit circles. For all transformation nodes, clearly indicate what transformation is being performed. [Hint: All your geometry nodes must be at the leaves of the tree.]

   (b) Suppose the propellor *linearly* accelerates from 0 to N rotations/second in one second. We draw the propellor at n frames/second. That means we need to draw n frames during the acceleration. Please describe how to obtain the rotation angle for the i-th frame, $1 \leq i \leq n$. You can assume the first frame is drawn at time $\frac{1}{n}$.